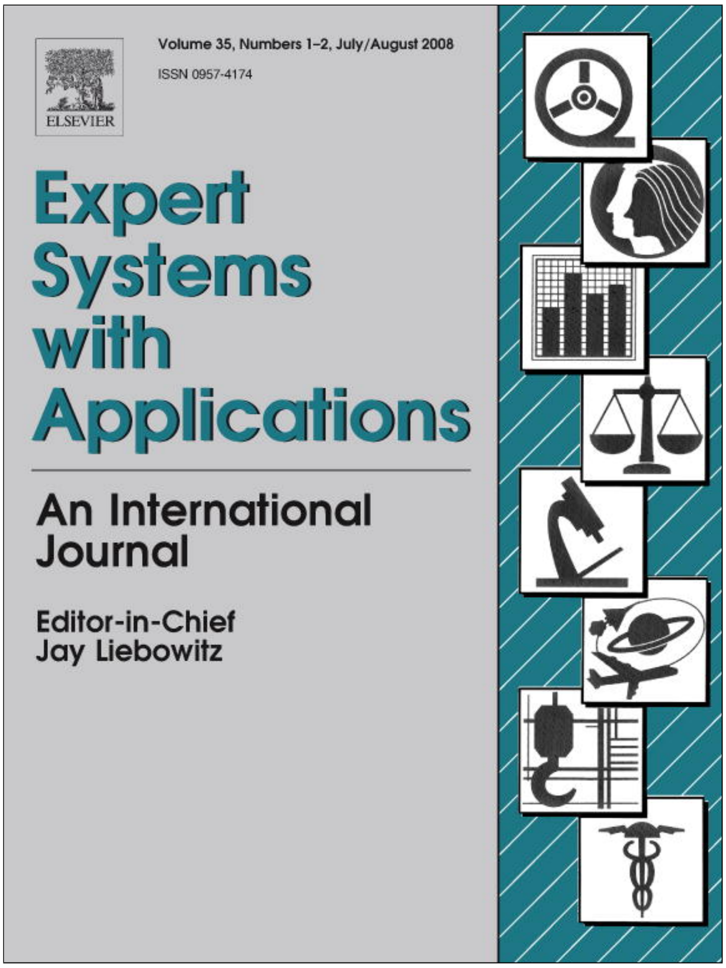
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Personal financial planning based on fuzzy multiple objective programming

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Abstract

Personal financial planning involves managing all the money activities during a planner's lifetime. Traditional personal financial planning procedures begin with the planner's financial status, goals, and expectations for the future before future cash flows of different time periods under various scenarios can be determined. If the planning results fail to meet the planner's expectation, the planner adjusts tunable parameters repeatedly until an acceptable financial arrangement can be obtained. Such a "trial-and-error approach" or "what-if analysis" does not promise to achieve the optimal plan while numerous outcomes burden the planner. Multiple objectives with different goals of different importance levels might be involved in this decision-making problem. Since the objectives tend to conflict with each other, this study proposes to solve the problem based on a decision model that incorporates a fuzzy multiple objective programming method to achieve better solutions than using "trial-and-error".

Keywords: Personal financial planning; Mathematical programming; Fuzzy multiple objective programming

1. Introduction

Personal financial planning (Madura, 2002) manages all money activities during a person's lifetime, including maximizing one's wealth, satisfying one's life goals, and managing risks from different sources. Financial planning begins with examining one's personal financial statements, requiring the planner to provide his/her financial status, life goals, risk preferences, and so on. The planner makes a better decision through trial calculations under different scenarios. Such a "what-if" analysis might present difficulties. First, solving the financial planning problem by trial-and-error gives a satisfying suggestion, but not necessarily the best that can be found. Although some numerical analysis methods, such as the goal seeking of Crabb (1999), could be used to find the optimal decision for problems with a single objective, they are unrealistic because personal financial planning often involves multiple decision objectives to be achieved at the same time. Second, the planners might have various preferences for different objectives. Personal preferences regarding objectives should be considered when conducting the analysis. Third, financial goals set by the planner might be flexible. The planner might prefer to provide an acceptable range for a goal instead of an exact value, for example, the lower and upper limits for house price. Finally, as Fortin (1997) have stated, the solution to a financial planning problem might not be in a closed-form.

In view of the above difficulties, mathematical programming appears promising, which motivates this study. Goal programming and compromise programming (Yu, 1985) have a long history in dealing with multiple objective optimization problems. However, they fail to handle flexible goals. The only known method capable of doing this is fuzzy goal programming, but traditional fuzzy goal programming methods cannot incorporate objective weights. Lin (2004) recently proposed a weighted max–min

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Nomenclature

l_0	initial living expense	d'	actual down payment
τ	the time to own a house	m	expected amortization payment
v	the value of the house	m'	actual amortization payment
p	the amount of pension	r_t	expected house rent
$\mathbf{x}$	vector of decision variables	r'_t	actual house rent
$f_i(\mathbf{x})$	objective function	s_t	salary income
$\mu_i(f_i)$	membership function (utility function) of $f_i(\mathbf{x})$	e_t	earning from investment
u_t	0–1 variable signifying the purchase of a house	l_t	living expense
y_t	0–1 variable indicating the ongoing of amortization schedule	A_t	investment position
		b_t	balance
z_t	0–1 variable indicating the cease of amortization schedule	k	annual rate of return
		I	inflation rate
T	financial planning horizon	g_i	goal value for $f_i(\mathbf{x})$
τ	the time to buy a house	$\underline{g}_i$	lower (upper) tolerance for $f_i(\mathbf{x})$
H	loan duration	λ	minimum achieved level among objective goals
d_t	expected down payment	w_i	weight of $f_i(\mathbf{x})$

approach for fuzzy goal programming problems, which method considers objective weights and appears to be the best choice for the personal financial planning problem.

This study formulates a decision model for financial planning that considers the incomes from salary and investment, and the expenses for living, purchasing a house, and raising children. Four objectives are considered, including the level of living expense, the time to buy a house, the value of the house, and the pension available at retirement. All the objectives that contribute to one's life quality before and after retirement are to be maximized except the time to buy a house. The decision model incorporates Lin's method (Lin, 2004) for fuzzy goal programming, so as to consider objective weights, which method is capable of giving more important objectives higher levels of goal achievement than less important ones. Numerical examples are provided to show the effectiveness of the proposed method. The rest of this paper is organized as follows. Section 2 illustrates the personal financial planning problem by an example, and formulates a decision model for that problem. Section 3 discusses methods for fuzzy goal programming, and gives a fuzzy goal programming model for the financial planning problem. Section 4 presents the numerical examples, and finally, Section 5 draws conclusions.

2. Personal financial planning

To illustrate the personal financial planning problem, consider the following scenario.

Mr. Chiang is a 30 years old white-collar worker whose wife is 28 years old. The couple plan to have their first baby after 2 years and have another after 5 years. Mr. Chiang's annual salary is about NT600,000, and Mrs. Chiang's is 400,000. Living expense for this couple is about 450,000 per year, and they pay 200,000 for house rent every year before they have their own house. Their incomes and expenses are expected to increase with the inflation rate, which is estimated to be 2%. They have currently 500,000 in investment with an annual rate of return 5%, and all surplus incomes would be joined to the investment. Each child will increase the living expense by 25% before they are graduated from university at age 22. Mr. and Mrs. Chiang wishes they can possess their own house at a cost of about 8,000,000 in 10 years. Besides the down-payment that is required by the bank to be at least 20% of the house value, they intend to pay the deficit through an amortization schedule of 20 years with an annual interest rate of 8%. The couple plan to retire when Mr. Chiang is 60. The retirement payment will be the double of their annual salaries at that time.

The decision variables that Mr. Chiang can control in this typical personal financial planning problem are assumed to be the living expense, the time to purchase their house, the price of the house and consequently their pension at retirement. Other variables are uncontrollable environmental parameters. The decision they made directly influences the family's overall life quality. The planner wishes to achieve as high living expense, house price and pension as possible, but to own the house as early as possible. The objectives conflict with each other because shortage of cash is not allowed by Mr. Chiang when all expenses must be paid by their liquid assets. For examples, raised living expense gives a better life, but the results are very likely to be delayed house ownership, a cheaper house, or lesser pension at retirement. Consequently, Mr. Chiang is confronting a typical multiple objective decision-making problem. A trial-and-error method might be helpful, but it is very exhausting. Besides, the optimal solution might never be found. Mathematical programming appears to be the only way of solving this financial planning problem. However, a model for this problem must be built before a mathematical programming method can be used to solve the problem.

Let l_0, τ, v and p denote the initial living expense, the time to own a house, the value of the house, and the desired pension, respectively. Also, let $\mu_1(l)$, $\mu_2(\tau)$, $\mu_3(v)$ and $\mu_4(p)$ be the utility functions for these items, respectively. The objective for this decision-making problem is thus to maximize all the utility functions, and the decision model can be formulated as follows.

Model 1.

$$
\begin{aligned}
&\text{Max} && \mu_1(l_0) && (1)\\
&\text{Max} && \mu_2(\tau) && (2)\\
&\text{Max} && \mu_3(v) && (3)\\
&\text{Max} && \mu_4(p) && (4)\\
&\text{subject to} && \tau \geqslant t - M(1-u_t), \quad \text{for } t = 1,\ldots,T, && (5)\\
& && \tau \leqslant t + M(1-u_t), \quad \text{for } t = 1,\ldots,T, && (6)\\
& && \sum_{t=1}^{T} u_t = 1, && (7)\\
& && My_t \geqslant t - \tau + 1, \quad \text{for } t = 1,\ldots,T, && (8)\\
& && M(1-y_t) \geqslant \tau - t, \quad \text{for } t = 1,\ldots,T, && (9)\\
& && Mz_t \geqslant t - (\tau + H - 1), \quad \text{for } t = 1,\ldots,T, && (10)\\
& && M(1-z_t) \geqslant (\tau + H) - t, \\
& && \text{for } t = 1,\ldots,T, && (11)\\
& && r'_t \geqslant r_t(1-y_t), \quad \text{for } t = 1,\ldots,T, && (12)\\
& && M(1-y_t+z_t) + m' \geqslant m, \\
& && \text{for } t = 1,\ldots,T, && (13)\\
& && m + M(1-y_t+z_t) \geqslant m', \\
& && \text{for } t = 1,\ldots,T, && (14)\\
& && M(y_t - z_t) \geqslant m', \quad \text{for } t = 1,\ldots,T && (15)\\
& && M(1-u_t) + d'_t \geqslant d, \quad \text{for } t = 1,\ldots,T && (16)\\
& && Mu_t \geqslant d'_t, \quad \text{for } t = 1,\ldots,T && (17)\\
& && s_t + e_t - l_t - r'_t - d'_t - m' = b_t, \\
& && \text{for } t = 1,\ldots,T, && (18)\\
& && s_t = s_{t-1}(1+I), l_t = l_{t-1}(1+I), \\
& && r_t = r_{t-1}(1+I), \quad \text{for } t = 1,\ldots,T, && (19)\\
& && e_t = A_{t-1}k, \quad \text{for } t = 1,\ldots,T, && (20)\\
& && A_t = A_{t-1} + b_{t-1}, \quad \text{for } t = 1,\ldots,T, && (21)\\
& && m = (v-d)\text{PVIFA}, && (22)\\
& && p = A_T + b_T + 2s_T, && (23)\\
& && b_t \in R, \quad u_t, y_t, z_t = 0, 1, \quad \text{for } t = 1,\ldots,T,
\end{aligned}
$$

where T is the planning horizon.

To determine when to buy the house, let 0–1 variable u_t signify the purchase of house. $u_t = 1$ means that a house is bought in year t such that the down payment d is incurred; otherwise, $u_t = 0$. Eqs. (5) and (6) are used to find out when the house is bought, where M represents a very large positive number. The two constraints take effect when $u_t = 1$ such that τ must equal t; otherwise, $u_t = 0$ causes the two constraints to be ineffective. Eq. (7) means that house purchase can happen only once.

0–1 variable y_t represents whether the planner has owned a house or not, and z_t signifies whether the amortization schedule has ended or not. $y_t = 1$ means that the house has been bought, and $z_t = 1$ means that the amortization schedule has already ended. Eq. (8) requires that $y_t = 1$ when $t \geqslant \tau$, and Eq. (9) requires that y_t can only be 0 when $t < \tau$. Similarly, Eq. (10) requires that $z_t = 1$ when $t \geqslant \tau + H - 1$, where H is the loan duration. The condition $t \geqslant \tau + H - 1$ means that the amortization schedule has ended. On the other hand, when $t < \tau + H$, the loan is still going on such that z_t can only be 0 as required by Eq. (11), and the amortization must be paid.

Eqs. (12)–(15) determine the incurred house rent or amortization payment for each year. r_t is the expected house rent, while r'_t is the actually paid house rent, which is incurred only when $y_t = 0$. Similarly, m represents the expected amortization payment, and m' the actual one. Eqs. (13) and (14) require that $m' = m$ when $y_t = 1$ and $z_t = 0$. Otherwise, when $y_t = z_t = 0$ or 1, these two equations become ineffective, and Eq. (15) requires that m' be 0. In this model the down payment for the house is assumed to be variable. However, it must be at least 20% of the house value. Constraints (16) and (17) require that actual down payment d'_t equals the expected down payment d when $u_t = 1$; otherwise, $d'_t = 0$.

Eq. (18) balances each year's terminal cash flow. Notations s_t, e_t and l_t denote the salary income, earning from investment, and living expense in year t. Eq. (19) states that the salary, expenses and rent grow with the inflation rate I. In Eq. (20), A_t represents the investment position in year t, and k is the annual rate of return. Finally, the pension at the end of the planning horizon is the sum of A_T, b_T and $2s_T$. Notably, b_t can be negative but A_t cannot if the planner would not hold any debt other than the house amortization. The expense for raising children dose not appear in the model because they are included in l_t and have to be considered only when implementing the model. Given goal values for the objectives, the utility functions can be formulated as membership functions, and the multiobjective personal financial planning problem can be solved using a fuzzy goal programming method stated in the next section.

3. Fuzzy goal programming

Since Narasimhan (1980) first applied fuzzy set theory to goal programming, many achievements in the field of fuzzy goal programming (FGP) have been reported. Consider the following FGP problem with m fuzzy goals.

Model 2.

$$
\begin{aligned}
&\text{Find} && \mathbf{x} && (24)\\
&\text{to satisfy} && f_i(\mathbf{x}) \tilde{\geqslant} g_i, \quad i = 1,2,\ldots,n, && (25)\\
&\text{subject to} && \mathbf{x} \in F, && (26)
\end{aligned}
$$

where $\mathbf{x}$ is the solution in vector form, and F is the set of feasible solutions. The relation $\tilde{\geqslant}$ means that the left-hand side is *fuzzily* larger than the right-hand side. Fuzzily larger means that the achieved objective value is allowed to be less than the goal value, if full achievement is unattainable. Any FGP problem can be expressed with this model without loss of generality because every $\tilde{\leqslant}$ constraint can be converted to an equivalent $\tilde{\geqslant}$ constraint, and every $\cong$ constraint can be replaced by a equivalent couple of $\tilde{\leqslant}$ and $\tilde{\geqslant}$ constraints. Since all objectives might not be achieved simultaneously by any feasible solution, the decision maker may define a lower tolerance limit and accordingly a membership function for each objective to determine the achieved level of that objective. A membership function $\mu_i(f_i)$ for the ith fuzzy goal $f_i(\mathbf{x})\tilde{\geqslant}g_i$ can be expressed as,

$$\mu_i(f_i)=\begin{cases}1 & \text{if } g_i\leqslant f_i(\mathbf{x}),\\ \frac{f_i(\mathbf{x})-\underline{g}_i}{g_i-\underline{g}_i} & \text{if } \underline{g}_i\leqslant f_i(\mathbf{x})<g_i,\\ 0 & \text{if } f_i(\mathbf{x})\leqslant \underline{g}_i,\end{cases}\tag{27}$$

where $\underline{g}_i$ is a lower tolerance limit for the fuzzy goal $f_i(\mathbf{x})\tilde{\geqslant}g_i$.

Zimmermann (1978) first used the max–min operator of Bellman and Zadeh (1970) to solve fuzzy multiobjective linear programming problems. Most FGP methods follow the max–min approach. With Zimmermann's approach, using max–min as the operator, a max–min model for Problem (1) can be stated as follows:

Model 3.

$$\text{Max}\quad \lambda \tag{28}$$

$$\text{subject to}\quad \lambda\leqslant\frac{f_i(\mathbf{x})-\underline{g}_i}{g_i-\underline{g}_i},\quad i=1,2,\ldots,n, \tag{29}$$

$$\mathbf{x}\in F. \tag{26}$$

Several approaches have been developed to handle cases where objectives are not equally important. The first is the fuzzy weights approach of Narasimhan (1981), in which membership functions that represent linguistic priorities are defined on goal values. Strictly speaking, the fuzzy weights represent only the relative importance of the goal values of a certain objective rather than the relative importance among different objectives. The second is the weighted model considered by Hannan (1981), in which objectives are differently weighted to represent their relative importance, and the weighted sum of the deviations from the centers of triangular membership functions is minimized. However, this method uses only isosceles triangular membership functions. The third method is the preemptive structure of Tiwari, Dharmar, and Rao (1986). Like all preemptive structures, the shortcoming of this method is that higher-level objectives must be achieved before lower-level objectives can be considered. Higher-level objectives are thus infinitely more important than lower-level objectives. Objectives can only be either of equal importance or of extremely different importance. The additive model of Tiwari, Dharmar, and Rao (1987) also considers relative objective weights. However, Lin (2004) has indicated that the objective achievements by using weighted sum of objective functions do not correspond to the objective weights. When the DM provides relative weights for fuzzy goals with corresponding membership functions, the ratio of the achieved levels should be as close to the ratio of the objective weights as possible to reflect their relative importance. The additive model of Tiwari et al. does not necessarily give objectives of heavy weight higher achieved levels than others. Thus, Lin (2004) proposed a weighted max–min model as follows.

Model 4.

$$\text{Max}\quad \lambda \tag{28}$$

$$\text{subject to}\quad w_i\lambda\leqslant\frac{f_i(\mathbf{x})-\underline{g}_i}{g_i-\underline{g}_i},\quad i=1,2,\ldots,n, \tag{30}$$

$$\mathbf{x}\in F. \tag{26}$$

The problem formulated by Model 1 is now used to illustrate how to apply Model 4 to the personal financial planning problem. Restated, the decision variables are the initial living expense, the time to own a house, the value of the house, and the pension, namely, l_0, τ, v and p. In this particular decision-making problem, the objectives are precisely the variables themselves. Before solving the personal financial planning problem with the fuzzy goal programming approach, let $\mathbf{x}=(x_1,x_2,x_3,x_4)=(l_0,\tau,v,p)$, $f_1(\mathbf{x})=x_1=l_0$, $f_2(\mathbf{x})=x_2=\tau$, $f_3(\mathbf{x})=x_3=v$, and $f_4(\mathbf{x})=x_4=p$. Applying the weighted max–min approach to Model 1 with the numerical data described in the financial planning scenario leads to the following auxiliary model.

Model 5.

$$\text{Max}\quad \lambda \tag{28}$$

$$\text{subject to}\quad w_1\lambda\leqslant\frac{l_0-\underline{g}_1}{g_1-\underline{g}_1}, \tag{31}$$

$$w_2\lambda\leqslant\frac{\tau-\underline{g}_2}{g_2-\underline{g}_2}, \tag{32}$$

$$w_3\lambda\leqslant\frac{v-\underline{g}_3}{g_3-\underline{g}_3}, \tag{33}$$

$$w_4\lambda\leqslant\frac{p-\underline{g}_4}{g_4-\underline{g}_4}, \tag{34}$$

$$r_0=200,000,\quad I=5\%,\quad s_0=1,000,000,$$
$$H=20,\quad A_0=500,000,$$
$$\text{PVIFA}=0.10185,$$
$$(5)\text{–}(23).$$

Numerical examples in the next section will show how to use this auxiliary model to solve the personal financial planning problem.

4. Numerical examples

Based on the case scenario stated in Section 2, the parameters are $r_0=200{,}000$, $I=2\%$, $s_0=1{,}000{,}000$,

$H = 20$, $A_0 = 500{,}000$, and PVIFA = 0.10185. One way of using Model 5 is to solve the problem based on objective goals provided by the planner. First, assume that the planner has provided the satisfied and tolerable living expenses to be 400,000 and 300,000, respectively, leading to the membership function for living expense as

$$\mu_1(f_1) = \begin{cases} 1 & \text{if } 400,000 \leqslant l_0, \\ \frac{l_0 - 300,000}{400,000-300,000} & \text{if } 300,000 \leqslant l_0 < 400,000, \\ 0 & \text{if } l_0 \leqslant 300,000. \end{cases} \tag{35}$$

Also, the planner hopes that the house can be bought in 10 years but had better in 8 years with a cost between 5,000,000 and 7,500,000, giving the membership function for the time to buy a house, and that for the house value as

$$\mu_2(f_2) = \begin{cases} 1 & \text{if } 8 \geqslant \tau, \\ \frac{\tau-10}{8-10} & \text{if } 10 > \tau \geqslant 8, \\ 0 & \text{if } \tau \geqslant 10, \end{cases} \tag{36}$$

and

$$\mu_3(f_3) = \begin{cases} 1 & \text{if } 7,500,000 \leqslant v, \\ \frac{v-5,000,000}{7,500,000-5,000,000} & \text{if } 5,000,000 \leqslant v < 7,500,000, \\ 0 & \text{if } v \leqslant 5,000,000. \end{cases} \tag{37}$$

Finally, the planner wishes that the pension can be as large as 10,000,000 and at least 8,000,000. Therefore, the membership function for the pension is

$$\mu_4(f_4) = \begin{cases} 1 & \text{if } 10,000,000 \leqslant p, \\ \frac{p-80,00,000}{10,000,000-80,00,000} & \text{if } 8,000,000 \leqslant p < 10,000,000, \\ 0 & \text{if } p \leqslant 8,000,000. \end{cases} \tag{38}$$

Table 1
Optimal solution for each objective

	$l(0)$	τ	v	p
$\mathbf{x}_1^*$	514,825	11	5,000,000	8,304,650
$\mathbf{x}_2^*$	436,305	3	5,000,000	15,437,981
$\mathbf{x}_3^*$	300,000	11	13,059,464	8,319,725
$\mathbf{x}_4^*$	300,000	10	5,000,000	38,446,839
$f_i^* = g_i$	514,825	3	13,059,464	38,446,839
$f_i^- = \underline{g}_i$	300,000	11	5,000,000	8,304,650

Next, assume that the objectives have equal weights. That is $\mathbf{w} = (0.25, 0.25, 0.25, 0.25)$. The optimal solution obtained by solving model 5 is $l(0) = 405{,}358$, $\tau = 8$, $v = 7{,}633{,}951$, and $p = 10{,}716{,}575$. With these values, the corresponding goal achievement levels are respectively $\mu_1 = 1.054$, $\mu_2 = 1.500$, $\mu_3 = 1.054$, and $\mu_4 = 1.358$. The results show that each achieved goal value exceeds its expected value, indicating that the planner is too conservative. However, a planner often has difficulty in determining appropriate goal values. For example, assume that the goal for buying a house is $g_2 = 2$. The optimal solution becomes $l(0) = 366{,}667$, $\tau = 5$, $v = 7{,}004{,}129$, and $p = 14{,}904{,}734$. The achievement levels are $\mu_1 = 0.667$, $\mu_2 = 0.667$, $\mu_3 = 0.802$, and $\mu_4 = 1.000$. The achieved level for the first goal is only 2/3, while the achieved value for the fourth goal is far beyond required. In fact, buying a house before year 3 is literally impossible to achieve. An alternative approach can be used to determine appropriate goal values to overcome this difficulty.

Let $\mathbf{x}_i^*$ be the optimal solution for objective $f_i(\mathbf{x})$. Then the anti-ideal solution $\mathbf{x}_i^-$ for $f_i(\mathbf{x})$ can be defined as

$$f_i(\mathbf{x}_i^-) = \min\{f_i(\mathbf{x}_k^*), k = 1, \ldots, n\}, \tag{39}$$

assuming that all objectives are to be maximized. For objective $f_i(\mathbf{x})$, one can then use the optimal value $f_i^*(\mathbf{x}_i^*)$ and the anti-optimal value $f_i^-(\mathbf{x}_i^-)$ as the goal and the tolerance limit, respectively. Table 1 shows the optimal solutions for the objectives, and therefrom the goals and the tolerance limits.

The membership function can be redefined based on the goals and the tolerance limits listed in Table 1. Again, assuming equal weights, the optimal solution becomes $l(0) = 366{,}025$, $\tau = 8$, $v = 7{,}477{,}022$, and $p = 17{,}568{,}649$. The corresponding goal achievement levels are respectively $\mu_1 = 0.307$, $\mu_2 = 0.375$, $\mu_3 = 0.307$, and $\mu_4 = 0.307$. Theoretically, the achieved level should be equal. The achieved level for μ_2 is larger than others because τ must be an integer. Table 2 shows different solutions that can be obtained when the planner changes the weights to express different importance levels of objectives. The achieved levels approximately proportionate to the objective weights. Similarly, the slight difference is due to the constraints and that τ must be an integer. The obtained solutions can be easily verified using a spreadsheet software. For example, Table 3 lists the cash flows in the planning horizon for $\mathbf{w} = (0.25, 0.25, 0.25, 0.25)$, with a down payment of 3,191,477. Other constraints can be included into the

Table 2
Optimal solution with respect to different objective weights

Weight vector	$l(0)$ [μ_1]	τ [μ_2]	v [μ_3]	p [μ_4]
$\mathbf{w} = (0.25, 0.25, 0.25, 0.25)$	366,025 [0.307]	8 [0.375]	7,477,022 [0.307]	17,568,649 [0.307]
$\mathbf{w} = (0.70, 0.10, 0.10, 0.10)$	467,050 [0.778]	10 [0.125]	5,895,303 [0.111]	11,653,060 [0.111]
$\mathbf{w} = (0.10, 0.70, 0.10, 0.10)$	326,853 [0.125]	4 [0.875]	6,906,728 [0.237]	19,277,901 [0.364]
$\mathbf{w} = (0.10, 0.10, 0.70, 0.10)$	323,115 [0.108]	10 [0.125]	11,070,360 [0.753]	11,547,934 [0.108]
$\mathbf{w} = (0.10, 0.10, 0.10, 0.70)$	324,218 [0.113]	10 [0.125]	5,908,581 [0.113]	32,091,139 [0.789]
$\mathbf{w} = (0.40, 0.30, 0.20, 0.10)$	408,983 [0.507]	7 [0.500]	7,044,328 [0.254]	12,127,518 [0.127]

Table 3
Cash flows in the planning horizon for equal objective weights

Year	Salary income	Living expense	Earning from investment	Rent	Amortization payment	Balance	Investment position
0	1,000,000	366,025	0	0	0	0	500,000
1	1,020,000	373,346	25,000	204,000	0	467,654	967,654
2	1,040,400	380,812	48,383	208,080	0	404,687	1,372,342
3	1,061,208	388,429	68,617	212,242	0	432,048	1,804,389
4	1,082,432	396,197	90,219	216,486	0	361,869	2,166,259
5	1,104,081	404,121	108,313	220,816	0	385,396	2,551,654
6	1,126,162	412,204	127,583	225,232	0	410,207	2,961,862
7	1,148,686	420,448	148,093	0	436,492	−2,961,862	0
8	1,171,659	428,857	0	0	436,492	91,882	91,882
9	1,195,093	437,434	4594	0	436,492	107,044	198,926
10	1,218,994	446,182	9946	0	436,492	123,175	322,101
11	1,243,374	455,106	16,105	0	436,492	140,328	462,428
12	1,268,242	464,208	23,121	0	436,492	158,559	620,987
13	1,293,607	473,492	31,049	0	436,492	177,925	798,912
14	1,319,479	482,962	39,946	0	436,492	198,489	997,401
15	1,345,868	492,621	49,870	0	436,492	220,314	1,217,715
16	1,372,786	502,474	60,886	0	436,492	243,468	1,461,183
17	1,400,241	512,523	73,059	0	436,492	268,023	1,729,206
18	1,428,246	522,774	86,460	0	436,492	294,053	2,023,260
19	1,456,811	533,229	101,163	0	436,492	321,638	2,344,898
20	1,485,947	543,894	117,245	0	436,492	350,859	2,695,757
21	1,515,666	554,772	134,788	0	436,492	381,804	3,077,561
22	1,545,980	565,867	153,878	0	436,492	414,565	3,492,126
23	1,576,899	577,185	174,606	0	436,492	449,236	3,941,362
24	1,608,437	588,728	197,068	0	436,492	633,103	4,574,465
25	1,640,606	600,503	228,723	0	436,492	682,208	5,256,673
26	1,673,418	612,513	262,834	0	436,492	887,247	6,143,920
27	1,706,886	624,763	307,196	0	0	1,389,319	7,533,239
28	1,741,024	637,258	376,662	0	0	1,480,428	9,013,667
29	1,775,845	650,004	450,683	0	0	1,576,524	10,590,191
30	1,811,362	663,004	529,510	0	0	1,677,867	12,268,059

decision model, such as an upper limit for the down payment.

5. Conclusions

Personal financial planning involves managing all money activities during a planner's lifetime. Often a ''trial-and-error approach'' or ''what-if analysis'' is required to reach an acceptable financial arrangement to meet the planner's expectation. However, such a method does not promise to achieve the optimal plan. Besides, the planner is burdened with numerous analytical outcomes. Conflicting objectives with different goals of different importance levels might be involved in this decision-making problem. Since decision objectives have different units and scales, traditional methods for multiple objective optimization, such as goal programming, suffer from the problem of incommensurability. Therefore, this study proposed a decision model for the personal financial planning problem, and applied a fuzzy goal programming method to solve it and to achieve better solutions than traditional methods. Numerical examples are provided to show the method's effectiveness. The method helps to compromise among the objectives, and is capable of giving a more important objective a higher level of goal achievement than less important objectives. Although the proposed model uses many 0–1 variables, it is highly efficient, for a common problem can be solved within a few seconds.

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