The Journal of Risk and Insurance

© *The Journal of Risk and Insurance*, 2010, Vol. 77, No. 2, 399-422
DOI: 10.1111/j.1539-6975.2009.01326.x

Estimation of Housing Price Jump Risks and Their Impact on the Valuation of Mortgage Insurance Contracts

Ming-Chi Chen
Chia-Chien Chang
Shih-Kuei Lin
So-De Shyu

Abstract

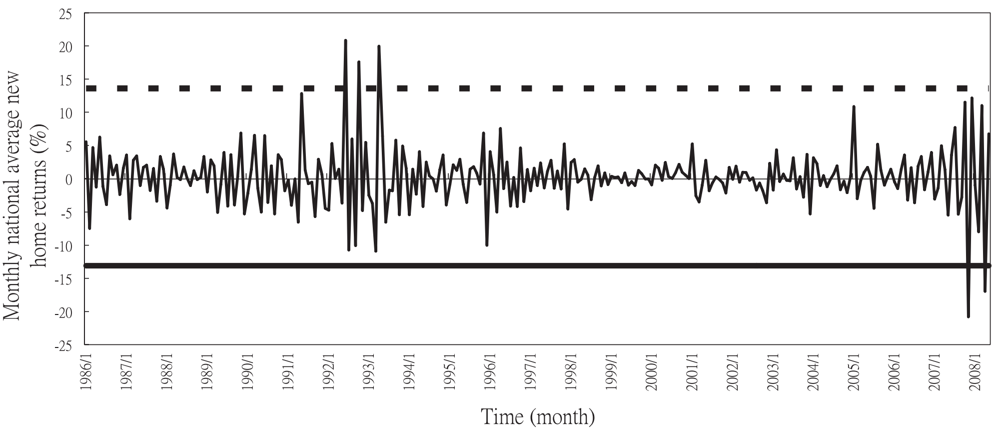
Housing price jump risk and the subprime crisis have drawn more attention to the precise estimation of mortgage insurance premiums. This study derives the pricing formula for mortgage insurance premiums by assuming that the housing price process follows the jump diffusion process, capturing important characteristics of abnormal shock events. This assumption is consistent with the empirical observation of the U.S. monthly national average new home returns from 1986 to 2008. Furthermore, we investigate the impact of price jump risk on mortgage insurance premiums from shock frequency of the abnormal events, abnormal mean and volatility of jump size, and normal volatility. Empirical results indicate that the abnormal volatility of jump size has the most significant impact on mortgage insurance premiums.

Introduction

Mortgage insurance has an important role in the housing finance market since it transfers the borrower's default risk exposure from the lenders to insurers and facilitates the creation of secondary mortgage markets (see Canner and Passmore, 1994). When determining mortgage termination, the present value of amortizing mortgage payments and the ability of the borrower to release from the payments through either prepayment or default must be considered. Although the prepayment decision is significantly affected by the interest rate, the house price influences significantly the

Ming-Chi Chen is from National Sun Yat-sen University, Kaohsiung, Taiwan. Chia-Chien Chang is from National Kaohsiung University of Applied Science, Kaohsiung, Taiwan. Shih-Kuei Lin is from National University of Kaohsiung, Kaohsiung, Taiwan. So-De Shyu is from National Sun Yat-sen University, Kaohsiung, Taiwan. Ming-Chi Chen can be contacted via e-mail: mcchen@finance.nsysu.edu.tw. The authors are grateful for the valuable comments of the anonymous referees.

FIGURE 1
U.S. National Average New Home Price Returns for Single-Family Mortgage



Note: The dashed (solid) line represents the mean of the housing price return plus (minus) three standard deviations.

decision to default.[1] Some studies show that changes in the loan-to-value ratio and housing price produce a wider range of mortgage default (see Kau et al., 1992; Kau, Keenan, and Muller, 1993; Kau and Keenan, 1995, 1996, 1999). When the loan-to-value ratio is higher, the price of mortgage insurance is higher. Furthermore, the house price volatility parameter is important for mortgage insurance and the impact of increasing the house price volatility is significant. Hence, the housing price change is a crucial factor in determining mortgage insurance premiums.[2]

In the previous literature, housing price change is assumed to follow a traditional geometric Brownian motion, and this assumption is reasonable under relatively stable housing prices. For example, all related studies on mortgage insurance pricing (see, e.g., Kau et al., 1992, 1995; Kau, Keenan, and Muller, 1993; Kau and Keenan, 1995, 1999; Bardhan et al., 2006) assume that the housing price process follows a geometric Brownian motion. However, Figure 1 shows the U.S. national average new home price returns for single-family mortgage from January 1986 to June 2008. It can be seen that there were 14 times when the monthly housing price changed more than 10 percent per month. In particular, the highest monthly housing price returns was 20.85 percent, in June 1992, while the lowest monthly housing price returns was −22.76 percent, in November 2007. Since 2007, with the higher interest rates and higher mortgage payments, subprime crisis occurred, which caused significantly downward jumps of

[1] Some empirical studies also indicate that the patterns of default and prepayment are significantly explained by the economic risk factors, such as interest rate, housing price return, loan-to-value ratio, and unemployment rate (Campbell and Dietrich, 1983; Schwartz and Torous, 1989, 1993; Quigley and Van Order, 1990, 1995; Deng, Quigley, and Van Order, 2000; Lambrecht, Perraudin, and Satchell, 2003; Caselli, Gatti, and Querci, 2008).

[2] Some empirical studies also examine the effects of catastrophic events on property values (see Bin, Kruse, and Landry, 2008).

housing price. On the other hand, other abnormal shocks, such as "Black Wednesday" in September 1992 or the "Iraq disarmament crisis" in July 1993, caused the U.S. Federal Reserve to adapt an expansionary monetary policy. Previous studies have suggested a strong connection between real interest rates and housing prices. Harris (1989), Abraham and Hendershott (1996), Englund and Ioannides (1997), Sutton (2002), Borio and Mcguire (2004), and Kostas and Zhu (2004), among others, all report the significantly negative relationship between the real interest rates and housing prices. Since the U.S. Federal Reserve lowered the interest rate 23 times from 1990 to 1992, we can understand that the announcement of lower interest rates could cause the U.S. housing price to make greater upward jumps. Overall, the housing price seems to have made higher jumps and volatility spikes during these years.

In order to properly model the housing price process, most of the discrete time models have been of the generalized autoregressive conditional heteroskedastic (GARCH) type, while the continuous time models were based on diffusion models. Mizrach (2008) and Schloemer et al. (2006) address the existence of jumps in housing markets. Mizrach demonstrates the jump risk component from the returns on the Chicago Mercantile Exchange (CME) futures. The empirical result indicates that, on average, it requires about 69 jump risks to be significant in the 315-day sample. Although GARCH models are capable of capturing smooth persistent changes in volatility, GARCH models are not suited to explaining the large discrete changes due to the abnormal events found in housing price returns. Hence, rather than studying volatility spikes, this article investigates the jump parameters of the housing price and their impacts on the mortgage insurance premium, when abnormal event information important to the housing market arrives, especially in the subprime mortgage crisis. Corresponding to the abnormal event of subprime mortgage crisis, the jump component of housing price represents systematic and nondiversifiable risk, which is, therefore, correlated with the market. To value the mortgage insurance contract, we use the Esscher transform technique developed by Gerber and Shiu (1994), which is a well-established technique in actuarial science and is suitable in cases where the log-returns of the underlying asset are governed by a process with independent and stationary increments. The behavior of the change of housing price can be divided into two parts: (1) continuous diffusion, which is responsible for the usual housing price movement and is described by a traditional Brownian motion, and (2) discontinuous jumps, which correspond to the arrival of new information important to the housing market.

This article contributes to the literature on mortgage insurance contract pricing in the following ways. First, this article estimates parameters of the jump diffusion model (JDM) using expectation-maximum (EM) gradient algorithms (based on the U.S. housing price data). EM gradient algorithms can be appropriate for latent-data problems because the total number of jumps for housing price is unobserved. The empirical results show that the likelihood ratio test (LRT) rejects the model without jumps at the significance level of 99 percent when using the national average for new home prices, but it does not reject the model without jumps when using the national average for previously occupied home prices. Second, to be consistent with the jump behavior of U.S. housing prices, this article applies a jump diffusion framework to derive the closed-form formula of mortgage insurance contracts by using Esscher transform technique. Our pricing formula for mortgage insurance contracts can also be reduced to the closed-form formula of Bardhan et al. (2006). Finally, to investigate how the jump risk of housing price impacts the valuation of mortgage insurance premiums, numerical analysis shows the relationships among mortgage insurance premium, the shock frequency of the abnormal event, the abnormal volatility of jump size, and the abnormal mean of jump size. If the shock frequency of the abnormal event (the abnormal volatility of jump size) increases two standard deviations, while all other parameters are fixed, the mortgage insurance premium should increase 0.27 percent (11.36 percent), respectively. Meanwhile, as the abnormal mean of jump size decreases two standard deviations, the mortgage insurance premium increases 6.57 percent. Therefore, the abnormal variance of jump size has the most significant effect on the mortgage insurance premium, and this implies the necessity of considering the jump parameters when pricing mortgage insurance contracts whose collateral asset is new homes.

The remainder of this article is organized as follows. The second section illustrates the mortgage insurance contract and the model. The third section derives the pricing formulas for mortgage insurance contracts under JDMs. Estimation via EM algorithms is shown in the fourth section. Empirical and numerical analyses are presented in the fifth section. The sixth section summarizes the article and gives conclusions. For simplicity, most proofs are in the Appendix.

THE CONTRACT AND MODEL

For the related models used in previous work, Kau et al. (1992, 1995), Kau, Keenan, and Muller (1993), Kau and Keenan (1995, 1999), and others consider two state variables: the interest rate and the housing price process. Prepayments and defaults are also typically determined endogenously within the model. However, implementation of these models requires complex numerical procedures since no closed-form formulas exist. Furthermore, some articles, such as Hendershott and Van Order (1987), also find evidence that mortgage insurance premiums are not very sensitive to interest rate volatility. Hence, Schwartz and Torous (1993), Dennis, Kuo, and Yang (1997), and Bardhan et al. (2006), among others, model the unconditional probability of default exogenously. Closest to our model is the option pricing method proposed by Bardhan et al. (2006). We develop a closed-form option-pricing framework for pricing mortgage insurance premiums and model the unconditional probability of default exogenously. Bardhan et al. assume that the housing price follows a geometric Brownian motion process. In contrast, to capture the jump phenomenon of housing price as shown in the Figure 1, the dynamic process of housing price proposed in this article combines a Brownian motion and a compound Poisson process. Therefore, in this section, the structure of mortgage insurance contract is illustrated and then we use the JDM to allow for abnormal shock events in housing prices.

Mortgage Insurance Contract

At time $t = 0$, i.e., at origination, the lender issues a T-month mortgage, secured by the housing, for the amount of $L(0) = L_V H(0)$. Let L_V be the initial loan-to-value ratio and $H(0)$ be the initial housing price. We assume that the mortgage loan has a fixed interest rate c and that installments x are paid monthly. Hence, with no prepayment or default prior to time t, the unpaid loan balance $L(t)$ at time $0 \leq t \leq T$ is given by the following expression:

$$L(t) = \frac{x}{c}\left(1 - \frac{1}{(1+c)^{T-t}}\right). \tag{1}$$

This equation shows that the unpaid loan balance is equal to the value of an ordinary annuity with a monthly payment equal to x and the discount rate equal to the contract rate c. In addition, at time $t = 0$, the insurer writes a mortgage insurance contract that promises to compensate the lender only when the borrower defaults. We follow the model of Bardhan et al. (2006) to consider that the realized loss for the insurer in case of the borrower's default can be represented as a portfolio of put options on the borrower's collateral. Thus, if a default occurs at time t, the insurer has to pay the lender the following amount:

$$LOSS(t) = \max\left[0, \min(L(t-) - H(t), L_R L(t-))\right], \tag{2}$$

where L_R denotes the loss ratio. Equation (2) implies that if the housing price exceeds the remaining loan balance, after the house is sold and the lender is compensated from the proceeds, the lender bears no loss, and hence, the loss for the insurer is also zero. On the other hand, if the housing price is not sufficient for a full repayment of the loan balance, the maximum loss to the insurer is equal to $L_R L(t-)$.

The previous studies on mortgage insurance pricing all assume that the housing price process follows a geometric Brownian motion. However, if the housing price process has an explicit jump risk that corresponds to the arrival of new important information to the housing market, the geometric Brownian motion will fail to capture important characteristics. Hence, it is necessary to use an appropriate model that considers jump risks to price the mortgage insurance contracts. In the following, we use jump diffusion framework to describe the housing price process with jump risks.

Model Structure of Housing Price With Jump Risks

We construct our model on a filtered probability space (Ω, F, P) generated by these two processes; i.e., the process of housing price $H(t)$, and the process of jump size in housing price $Y(t)$. There exists a unique physical probability P such that capital markets are complete. The filtration $\mathrm{F} = (F_t)_{t\geq 0}$ satisfies $F_t = F_t^W \vee F_t^Y$ for any time t, where $F_t^W = \sigma(W(u), 0 \leq u \leq t)$, and $F_t^Y = \sigma(Y(u), 0 \leq u \leq t)$. Hence, $F_t^W \vee F_t^Y$ contains complete information on Brownian motions and jump sizes of the housing price returns. Further, there are three types of agents in the economy: the lender, the borrower, and the insurer.

Assume that the risk-free security is the money market account $B(t) = e^{rt}$, where r is constant continuously compounded return, $r \in R^{++}$. Furthermore, let the process of the housing price be the combination of a Brownian motion and a compound Poisson process as follows:[3]

$$H(t) = H(0)\exp\left(\mu t + \sigma_H dW^P(t) + \sum_{n=0}^{N(t)} Y_n - \lambda E(Y)t\right), \tag{3}$$

where $H(0)$ is the initial housing price, μ is the expected growth rate of housing price, and σ_H is the constant volatility of the Brownian component of the housing price process. In addition, $W(t)$ is a standard Brownian motion. The role of $W^P(t)$ with drift can be used to capture the unanticipated instantaneous change of housing price, which is the reflection of normal events, but it may not work so well for abnormal shocks. For example, the government may suddenly increase interest rates, or a mortgage crisis may arise. Thus, a compound Poisson process is constructed here to address the total number of jumps and jump size corresponding to the arrival of abnormal information. $N(t)$ represents the total number of jumps (including the house price rise and drop event) during a time interval of $(0, t]$, and it is based on a Poisson process with a intensity parameter λ . Notation Y_n, $n = 1, 2, \ldots$ are i.i.d. random variables representing the size of the nth size of the jumps with the density function $f(dy)$ and the expectation $E(Y) < \infty$. In particular, we assume that the jump size is normal distributed with mean φ and variance δ^2, that is, $Y \sim N(\varphi, \delta^2)$. $\varphi > 0 (\varphi < 0)$ represents the upward average jump size (downward average jump size) in housing price in case of abnormal events during a time interval of $(0, t]$. In addition, all three sources of randomness, $W^P(t)$ standard Brownian motion, $N(t)$ Poisson process, and Y the jump size, are assumed to be independent.

Changes in the housing price have three components: (1) the expected instantaneous housing price change conditional on no abnormal events; (2) the unanticipated instantaneous housing price change, which is the reflection of causes that have a marginal impact on the gauge; and (3) the instantaneous change due to an abnormal shock event. The housing price $H(t)$ follows a geometric Brownian motion during time period $(0, t]$ given that no information of abnormal event arrivals during the time period. When information on an abnormal event arrives at time t, the housing price changes instantaneously from $H(t-)$ to $\exp(Y_n)\, H(t-)$.

Note that accurately predicting the unconditional probability of borrower default is not our purpose, and so we assume that the unconditional probability of borrower default at time $t \in T$ is exogenously determined and is set equal to $P(t)$.

MORTGAGE INSURANCE VALUATION

For the pricing methods for mortgage insurance contracts in the context of the literature, Kau et al. (1992, 1995), Kau, Keenan, and Muller (1993), and Kau and Keenan (1995, 1999) use arbitrage principles of option pricing theory to rationally price a mortgage. The value of the mortgage can be described as the solution to a partial differential equation in backward time, whose terminal and boundary conditions embody the terms of the contract. Dennis, Kuo, and Yang (1997) propose the actuarial

[3] This process is a special case of the Levy process (see Ballotta, 2005).

pricing method, in which a feasible premium structure is defined as one such that the present value of the expected loss (plus a gross margin) for the insurer is equal to that of the expected premium revenues. Bardhan et al. (2006) assume that the agents in the economy are risk neutral. In this case, the present value of the severity of loss would involve the expectation with respect to the risk neutral probability measure. Hence, they develop an option-pricing framework to price mortgage insurance contracts under the risk-neutral probability measure.

To compute a mortgage insurance contract, we also assume that financial markets are frictionless. There are no transaction costs or differential taxes, trading takes place continuously in time, borrowing and short selling are allowed without restriction and with full proceeds available, and borrowing and lending rates are equal. Furthermore, when the housing price process has jumps, the market becomes incomplete, and then there is no unique pricing measure. Because most shock events causing the housing price to jump, such as the subprime mortgage crisis, are systematic risks, the jump component of housing price represents both systematic and nondiversifiable risk. We make use of the Esscher transform measure developed by Gerber and Shiu (1994) to define the Radon-Nikodym process $\eta(t)$ as follows:

$$\eta(t) = \exp\left(-\frac{\sigma_H^2}{2}h^2 t + h\sigma_H W^P(t) + h\sum_{n=0}^{N(t)} Y_n - t\int_R (e^{hy}-1)f(dy)\right), \qquad (4)$$

where $\eta(t)$ is called the Esscher transform of parameter h. Hence, it is possible to select the risk-neutral Esscher measure as the measure P^h so that housing prices discounted at the risk-free rate are P^h-martingales. Under the risk neutral probability measure P^h, the dynamic process of housing price becomes:

$$H(t) = H(0)\exp\left\{\left(r - \frac{\sigma_H^2}{2} - \int_R e^{hy}(e^y-1)f(dy)\right)t + \sigma_H W^h(t) + \sum_{n=0}^{N(t)} Y_n\right\}. \qquad (5)$$

Under Equations (2) and (5), the present value of the severity of loss, $DL(t)$, can be given by the following expression:

$$\begin{aligned} DL(t) &= e^{-rt}E^Q[LOSS(t)] \\ &= e^{-rt}E^Q[\max(K_1 - H(t), 0)] - e^{-rt}E^Q[\max(K_2 - H(t), 0)], \end{aligned} \qquad (6)$$

where $K_1 = L(t-)$, $K_2 = (1 - L_R)L(t-)$.

When a borrower's default occurs at time $t \in T$, Equation (6) implies that the present value of the severity of loss, $DL(t)$, can be duplicated by a long position in a European put option with a strike price K_1 and a short position in a European put option with a strike price K_2, both with the time to maturity equal to the time to default t.

Based on the Esscher measure, the expectation in (6) can be represented, for all $t \in T$, as follows:[4]

$$DL(t) = \sum_{m=0}^{\infty} \frac{\exp(-\lambda\mu_{h+1}t)\,(\lambda\mu_{h+1}t)^m}{m!}\,[P(K_1) - P(K_2)], \qquad (7)$$

where $P(K_i) = K_i \exp(-r_{m;h}t)\Phi(-d_{2m}(K_i)) - H(0)\Phi(-d_{1m}(K_i))$, $i = 1, 2$,

$$d_{1m,2m}(K_i) = \frac{\ln(H(0)/K_i) + \left(r_{m;h} \pm \frac{\sigma_m^2}{2}\right)t}{\sqrt{\sigma_m^2 t}}, \quad r_{m;h} = r - \lambda(\mu_{h+1} - \mu_h) + \frac{m}{t}\ln\left(\frac{\mu_{h+1}}{\mu_h}\right),$$

$$\sigma_m^2 = \sigma_H^2 + \frac{m\delta^2}{t}$$

$$\mu_h = \exp\left(h\varphi + \frac{1}{2}h^2\delta^2\right), \quad \mu_{h+1} = \exp\left((h+1)\varphi + \frac{1}{2}(1+h)^2\delta^2\right).$$

Hence, volatility of the housing price process $\sigma_m^2\,t$ includes two components: normal volatility of the Brownian component $\sigma_H^2\,t$ during time t and abnormal volatility of jump size $m\delta^2$ when an abnormal event occurs m times. If $\lambda = 0$, this means that no abnormal shock event occurs, and so the volatility of housing price process captures only the normal volatility, and so Equation (7) can be reduced to the standard closed-form formula of Bardhan et al. (2006).

Since the housing price is independent of the unconditional probability of borrower default, the fair price (*FP*) of a mortgage insurance contract with jump risk is given as follows:

$$FP = \sum_{t=1}^{T} P(t)DL(t). \qquad (8)$$

Finally, across the mortgage life, the insurers are expected to earn a profit even though they may lose money in some periods. Thus we assume that the gross profit margin that the insurer requires is equal to q, and the mortgage insurance premium (*FPA*) with jump risk is given by the expression:

$$FPA = (1+q)FP. \qquad (9)$$

Equation (8) implies that the fair price (*FP*) is calculated by the summation of a series of the loss amount of the insurer if the borrower defaults in each month from inception to expiration. Hence, the insurer can judge in each month the probability that the borrower will default rather than at only maturity.

[4] The detailed proof of a European put option using the Esscher transform technique can be found in previous articles (e.g., Ballotta, 2005).

ESTIMATION VIA EM ALGORITHM

Using Equation (3), one can obtain the housing price $H(t+\Delta t)$ given as $H(t)$ resulting in

$$H(t+\Delta t)\mid F_t = H(t)\exp\left[\mu\Delta t + \sigma_H \Delta W(t) + \sum_{n>N(t)}^{N(t+\Delta t)} Y_n - \lambda\varphi\Delta t\right]. \tag{10}$$

We choose monthly time interval because monthly data are the highest frequency data for the housing prices. In each monthly time interval of length $\Delta t = 1$, we substitute $(\mu - \lambda\varphi)\Delta t$, $\sigma_H \Delta t$, $N(\Delta t)$ for $\mu - \lambda\varphi$, σ_H, N; hence, the rate of change for housing price $R_{\Delta t} = \ln(H(t+\Delta t)/H(t))$[5] can be written as:

$$\begin{aligned} R_{\Delta t} &= (\mu - \lambda\varphi)\Delta t + \sigma_H \Delta W_t + \sum_{n=1}^{N(\Delta t)} Y_n \\ &= \mu - \lambda\varphi + \sigma_H Z + \sum_{n=1}^{N} Y_n, \end{aligned} \tag{11}$$

where $Z \sim N(0,1)$.

To estimate parameters for the jump diffusion process, an estimation technique that can handle the latent component is needed because the total number of jumps for housing price are latent data. There is a wealth of estimation techniques available for models including a latent component, such as EM algorithm in Dempster, Laird, and Rubin (1977), Markov Chain Monte Carlo (MCMC) method in Eraker, Johannes, and Polson (2003), and the efficient method of moments (EMM) in Chernov et al. (2003).[6] EMM is fast and efficient but is not satisfactorily consistent and asymptotically normal. The EM-type algorithms, deterministic in nature, are designed to obtain the modes of posterior distributions in the maximum likelihood estimates (MLE) (see Tanner, 1996). Some studies, such as Nityasuddhi and Bohning (2003), Zeng and Cai (2005), and Kvarnstrom (2005), indicate that the MLE via the EM algorithm is strongly consistent and asymptotically normal. MCMC algorithms are stochastic and they iterate between simulating from the conditional distributions of latent data and the parameters, typically a more ambitious task and higher computation cost than the point estimation needed for the EM algorithm. In our model, the expressions for expectation in the first and the second partials of the MLE can be derived the closed-form formulae. Consequently, to reduce computation cost, this article uses EM gradient algorithms to estimate parameters for the jump diffusion process.

The EM algorithm is one of the pillars of modern computational statistics. To explain this algorithm, let us recall the conventions underlying the EM algorithm. Let

[5] The unit root test is used to confirm that the time series of the change rate of the housing price is stationary.

[6] Other estimation methods include simulated maximum likelihood as in Durham and Gallant (2002) and state-based generalized method of moments (GMM) as in Pan (2002).

$\tilde{R} = [R_1, R_2, \ldots, R_T]$ be the observed data and $\tilde{N} = [N_1, N_2, \ldots, N_T]$ be the latent data, and then the complete data are $\tilde{X} = [\tilde{R}, \tilde{N}] = [R_1, R_2, \ldots, R_T, N_1, N_2, \ldots, N_T]$. Although the statistician has control over how the complete data are defined, the sensible procedure is to choose $\tilde{X}$ so that it is trivial to estimate the parameters $\theta = (\mu, \sigma_H^2, \varphi, \delta^2, \lambda)$ of a model by maximum likelihood. To estimate the vector of unknown parameters $\theta = (\mu, \sigma_H^2, \varphi, \delta^2, \lambda)$ of the JDM, the EM algorithm starts from the initial guess value $\theta_0 = (\mu_0, \sigma_{H0}^2, \varphi_0, \delta_0^2, \lambda_0)$ and then iteratively in two steps: the E-step and the M-step.

E-Step

In the E-step of the EM algorithm, calculate the conditional expectation of the complete-data likelihood function $Q(\theta \mid \theta_\mathrm{n})$ given the data $\tilde{R}$ and current estimated value $\theta_\mathrm{n} = (\mu_n, \sigma_{Hn}^2, \varphi_n, \delta_n^2, \lambda_n)$ as follows:

$$\begin{aligned} Q(\theta \mid \theta_\mathrm{n}) &= E(\log L_C(\theta \mid \tilde{R}, \tilde{N}) \mid \tilde{R}, \theta_\mathrm{n}) = \sum_{t=1}^{T} E(\log L_C(\theta \mid R_t, N_t) \mid R_t, \theta_\mathrm{n}) \\ &= \sum_{t=1}^{T} \sum_{m_t=0}^{\infty} \Bigg[-\lambda + m_t \log \lambda - \log(m_t!) - \frac{1}{2} \log\left(2\pi\left(\sigma_H^2 + m_t \delta^2\right)\right) \\ &\qquad - \frac{(R_t - \mu - \lambda\varphi - m_t\varphi)^2}{2\left(\sigma_H^2 + m_t\delta^2\right)} \Bigg] P(N_t = m_t \mid R_t, \theta_\mathrm{n}). \end{aligned} \tag{12}$$

M-Step

In the M-step of the EM algorithm, compute the MLE of the parameters $\theta^1 = (\hat{\mu}, \hat{\sigma}_H^2, \hat{\varphi}, \hat{\delta}^2, \hat{\lambda})$ by maximizing $Q(\theta \mid \theta_\mathrm{n})$ found on the E-step:

$$\theta_{\mathrm{n}+1} = \arg\max_{\theta} Q(\theta \mid \theta_\mathrm{n}). \tag{13}$$

The essence of the EM algorithm is that increasing $Q(\theta \mid \theta_\mathrm{n})$ forces an increase in the log-likelihood of the observed data. Note that it is impossible to carry out the M-step exactly due to nonlinear equations. The fastest common algorithm for iteratively solving the M-step would be Newton's method, which has quadratic convergence, compared with the linear convergence in the EM algorithm. Hence, we use the EM gradient algorithm proposed by Lange (1995), in which a single iteration of Newton's method at each M-step would be adequate to ensure convergence of an approximate EM algorithm. Based on the EM gradient algorithm, the updating rule of the current parameter column vector θ_n is:

$$\theta_{\mathrm{n}+1} = \theta_\mathrm{n} - \left(d^{20} Q(\theta_\mathrm{n} \mid \theta_\mathrm{n})\right)^{-1} d^{10} Q(\theta_\mathrm{n} \mid \theta_\mathrm{n}). \tag{14}$$

The operators $d^{10}$and d^{20} take first and second partials with respect to the θ variables of $Q(\theta^n \mid \theta^n)$. The Hessian matrix $d^{20}Q(\theta_n \mid \theta_n)$ is indeed always negative and definite. This fact in turn implies strict concavity of $Q(\theta \mid \theta_n)$ and uniqueness of the maximum point θ_{n+1}. Therefore, in the M-step of nth iteration, one can derive the new parameter estimate θ_{n+1} by the Equation (14), and then θ_{n+1} is used to begin E-step of $(n+1)$th iteration. This two-step process is repeated until convergence within 1,000 iterations. The detailed proof of the log-likelihood function of JDM via EM gradient algorithm is shown in the Appendix.

Since the LRT has characteristics of invariance and consistency, we use the likelihood ratio statistic as follows:

$$\Lambda = 2\big(\ln L(\mathrm{R};\theta^1) - \ln L(\mathrm{R};\theta^*)\big), \tag{15}$$

where θ^1 is the MLE under the Poisson jump diffusion specification. θ^* is the parameter vector estimate corresponding to the local maximum when $\lambda = 0$ and hence when no jump structure is present. The null hypothesis is that the rate of change for housing prices is consistent with a lognormal diffusion process without a jump structure (the Black–Scholes model). On the other hand, the alternative hypothesis is that change rate of housing price stands for the JDM. Under the null hypothesis, Λ is asymptotically χ^2-distributed with two degrees of freedom.

Empirical and Numerical Results

Our data come from the Federal Housing Financial Board, which contains the terms of conventional single-family mortgage and national average homes prices in the United States. The categories of homes include previously occupied homes, new homes, and all homes, and the mortgage rates include both fixed and adjustable rates. We investigate and compare on a monthly basis previously occupied home price and new home prices with adjustable-rate mortgages. The contact interest rate, effective interest rate, term to maturity, mortgage loan amount, and loan-to-price ratio of previously occupied homes and of new homes are also collected for analysis. Our sample period is from January 1986 to June 2008, so that there are 270 observations for each variable.

Parameter Estimation and Jump Test

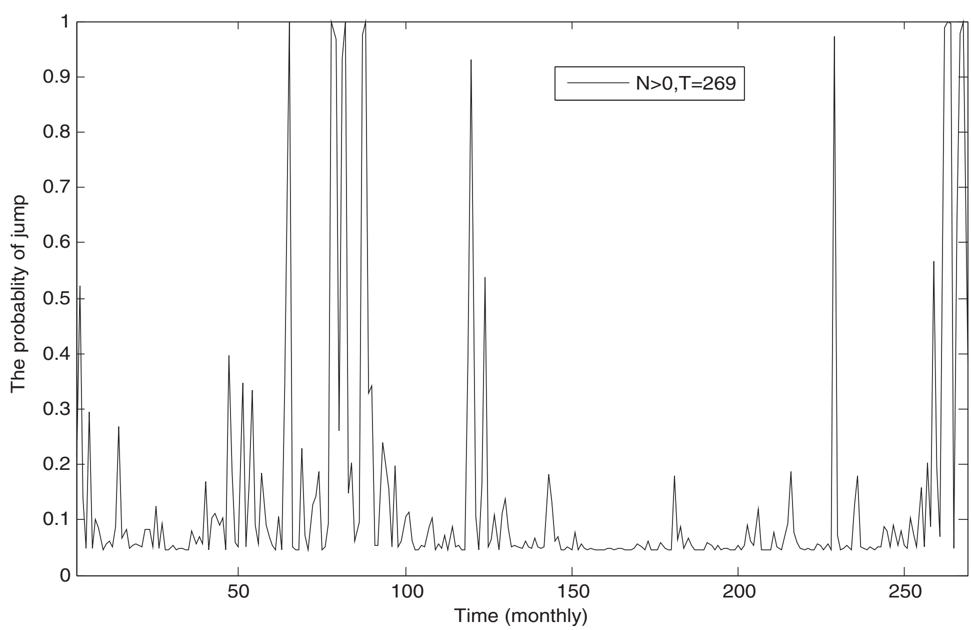
In Table 1, Panel A reports the parameters estimated from the JDM and Black–Scholes model (BSM) based on single-family mortgage national average new homes prices. By using EM gradient algorithms, the expected rate of change of the housing price, μ, is 1.2×10^{-3}, which implies that the change of the housing price increases by 1.2×10^{-3} per month on average. The positive sign of μ is consistent with the fact that U.S. housing prices rise over time. The abnormal mean, abnormal volatility, and shock frequency of the abnormal event are 2×10^{-2}, 7.9×10^{-3}, and 1.47×10^{-1}, respectively. The LRT statistics $\Lambda = 75.29$ reject the model without jumps at the significance level of 1 percent. Particularly, Figure 2 shows much higher jump probability of new homes prices (approach one) during 1992–1993 (the announcement of lowered interest rate) and 2007–2008 (mortgage subprime crisis). This result seems reasonable, implying that the average new home prices are sensitive to abnormal shock information or catastrophic events, such as the announcement of interest rate policy.

TABLE 1
Estimating and Testing the Model Based on U.S. National Average Home Price Returns for Single-Family Mortgage

Model	μ	σ_H^2	φ	δ^2	λ	LRT
Panel A: National Average New Homes Price Returns						
JDM	1.2×10^{-3}	8×10^{-4}	2×10^{-2}	7.9×10^{-3}	1.47×10^{-1}	$\Lambda = 75.29^*$
	(2.1×10^{-3})	(1×10^{-4})	(1.6×10^{-2})	(2.1×10^{-3})	$(5.4 \times 10^{-2})^*$	
BSM	2.8×10^{-3}	2×10^{-3}	–	–	–	
Panel B: National Average Previously Occupied Homes Price Returns						
JDM	2.1×10^{-3}	4.4×10^{-4}	-1.96×10^{-2}	4×10^{-5}	3.62×10^{-3}	$\Lambda = 0.037$
	(1.2×10^{-3})	(4×10^{-5})	(7.7×10^{-3})	(1.1×10^{-4})	(3.3×10^{-2})	
BSM	2.2×10^{-3}	4.9×10^{-4}	–	–	–	

*Denotes statistical significance at the 1 percent level, and the values in parentheses represent the standard deviation. JDM and BSM indicate jump diffusion model and Black–Scholes model, respectively.

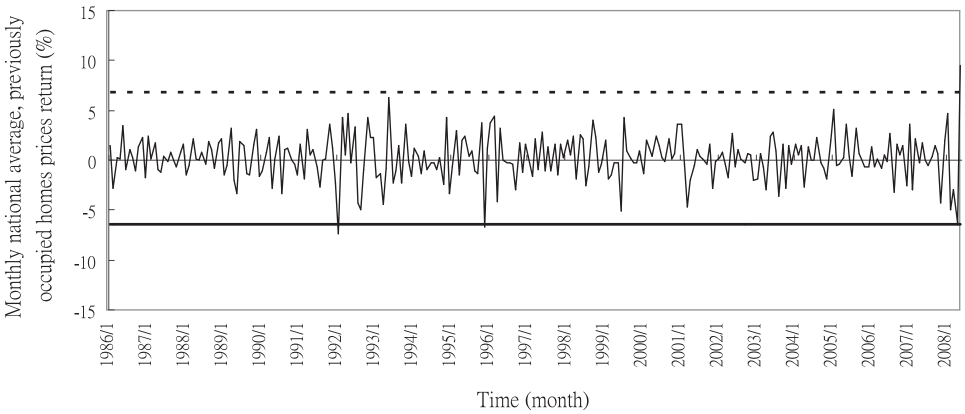
FIGURE 2
The Jump Probability of U.S. National Average New Home Price Returns for Single-Family Mortgage



Hence, the housing price has higher jump risk as abnormal shock information occurs.

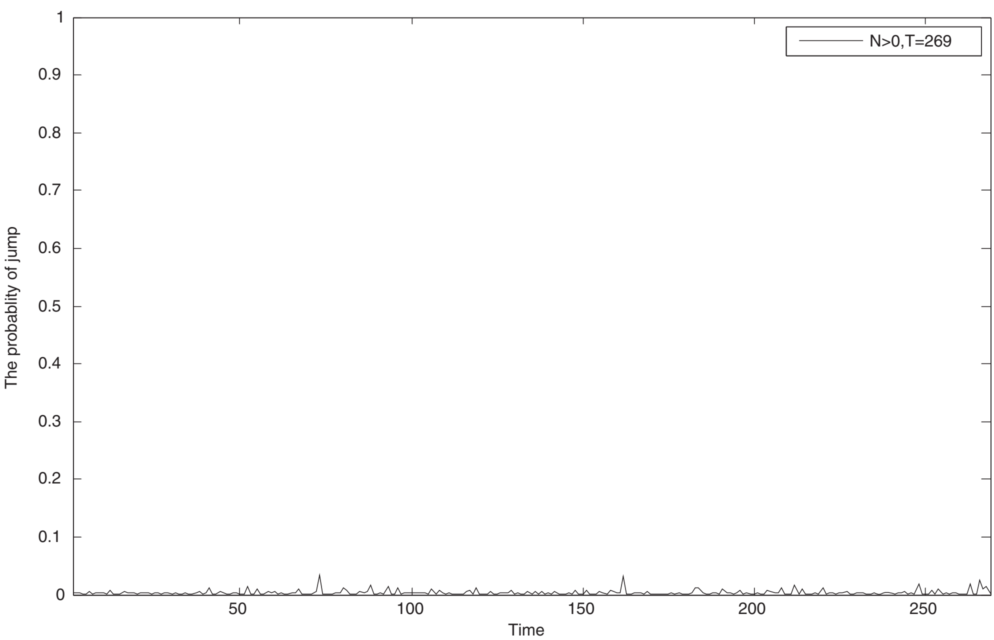
Panel B reports the parameters estimated for the previously occupied home prices. The LRT statistics, in which $\Lambda = 0.037$, do not reject the model without jumps at the significance level of 5 percent. If we examine the historical price movement of

FIGURE 3
U.S. National Average Previously Occupied Home Price Returns for Single-Family Mortgage



Note: The dashed (solid) line represents the mean of the housing price return plus (minus) three standard deviations.

FIGURE 4
The Jump Probability of U.S. National Average Previously Occupied Home Price Returns for Single-Family Mortgage



previously occupied homes in Figures 3 and 4, we can find only two times when the monthly housing price returns exceed three standard deviations, and thus the jump probability of the previously occupied homes is fairly small. In general, the previously occupied home prices appear to have little variation and so they may be less sensitive to changes in the economic situation, such as announcement of significant government policy. This could result from different degree of informational efficiency in these two markets. Typically, the new housing market tends to be more sensitive than existing market in terms of both quantities and price. This is because builders usually hold more information than individuals and can quickly respond to market conditions so that they can manipulate the supply quantities and prices in response to the changes in demand. Given that builders of new housing possess more information than individuals, these builders may use such information to their advantage, and in the process generate some short-term volatility in the market.

Note that there are other formal tests available for detection of jumps.[7] We use the adjusted ratio jump test statistics provided by Barndor-Nielsen and Shephard (2006), and compute the significant jump according to Andersen, Diebold, and Bollerslev (2007), finding that there are about 30 significant jump risks in the 269 monthly samples at a significance level of 5 percent based on the data of new home price.[8]

Numerical Analysis for Mortgage Insurance Premium

Base Parameters and Value of Mortgage Insurance Premiums. Results in this section are based on a set of parameters from the data and estimation of the national average new homes price returns for single-family mortgage, which was proved to have jump risk in the previous section. The base parameters and their standard errors of the normal volatility of the Brownian component (σ_H^2), the abnormal volatility of jump size (δ^2), the abnormal mean of jump size (φ), and shock frequency of the abnormal events (λ) on the mortgage insurance premium are taken from Table 1. The other base parameters are obtained by using the average value of observations from sample period. Table 2 presents these base parameter values on the mortgage insurance premium and the base value of the mortgage insurance premium is therefore calculated as \$2,289.6 by using Equations (6)–(8).

Do Jump Parameters Values Matter: Sensitivity Analysis. Table 3 further reports the sensitivity analysis of the mortgage insurance contract. The indicated parameters plus (minus) two standard deviations are used to demonstrate the abnormal volatility effect, the normal volatility effect, abnormal mean of jump size effect, and shock frequency effect, respectively. Hence, the parameters of the abnormal volatility of jump size are set to (3.7×10^{-3}, 7.9×10^{-3}), and 1.2×10^{-2}. From the sensitivity

[7] The formal statistical tests of jumps are provided by Barndor-Nielsen and Shephard (2006), Huang and Tauchen (2005), Eraker, Johannes, and Polson (2003), Andersen, Diebold, and Bollerslev (2007), etc.

[8] We also take the sample interval to be weekly changes, and follow Mizrach (2008) to compute 50-period rolling sample to analyze the statistical significance of the jump risk. The results represent about 70 jump risks is significant in the 1,075 weekly samples.

TABLE 2
Base Parameter Values of Mortgage Insurance Premium

$H = \$235,000$	Housing price
$L_R = 0.78$	Loss ratio
$r = 0.5\%$	Riskless interest rate[a]
$x = \$300$	Installments
$T = 360$ (30 year)	Term to maturity of mortgage contract
$c = 0.6\%$	Contract interest rate of mortgages
$q = 0.05$	Gross profit margin
$P(t) = 0.03$	Unconditional default probability

[a]The risk-neutral Esscher transform parameter h is determined by Esscher martingale condition (see Gerber and Shiu, 1994).

analysis, if the abnormal volatility of jump size increases, this causes the volatility of mortgage insurance premium to increase, and the Poisson probability to increase and the volatility of option (value of the severity of loss) to increase; then the value of mortgage insurance premium increases. Thus, with higher the abnormal volatility of jump size, the value of a mortgage insurance contract increases. This economic implication is that if there is a future crash in the housing market, housing prices will have an intense jump phenomenon and higher abnormal volatility of jump size. Based on this situation, as the abnormal volatility of jump size increases two standard deviations (from 7.9×10^{-3} to 1.2×10^{-2}), while all other parameters are fixed, the mortgage insurance premium should be \$2,550.0 rather than \$2,289.6. Thus, the mortgage insurance premium increases by 11.36 percent, implying that the insurer should collect higher premiums under the possibility if an event such as subprime mortgage crisis. On the other hand, if the abnormal volatility of jump size reduces two standard deviations (from 7.9×10^{-3} to 3.7×10^{-3}), while all other parameters are fixed, the mortgage insurance premium decreases to \$2,009.9 from \$2,289.6, a reduction of 12.23 percent.

Furthermore, the parameters of shock frequency of the abnormal event and the mean of jump size are respectively set to (9.3×10^{-2}, 1.47×10^{-1}, and 2.01×10^{-1}) and (-1.2×10^{-2}, 2×10^{-2}, and 5.2×10^{-2}). We find that the mortgage insurance premium is positively related to shock frequency of the abnormal events but negatively related to the mean of jump size. If a housing price crisis occurs and continues to deepen, increasing the possibility for a future housing recession, the frequencies abnormal events in the housing market could be predicted to rise. The mortgage insurance premium increases by 0.27 percent when the shock frequency of the abnormal events is predicted to increase two standard deviations. Furthermore, as the mean of jump size drops to -1.2×10^{-2} from 2×10^{-2}, the mortgage insurance premium increases by 6.57 percent. Similarly, the mortgage insurance premium decreases by 0.18 percent when the frequency of abnormal events of housing market

TABLE 3
Sensitivity Analysis of a Mortgage Insurance Contract

			λ		
			9.3×10^{-2}	1.47×10^{-1}	2.01×10^{-1}
-1.2×10^{-2}	3.7×10^{-3}	6×10^{-4}	2,085.4 (−8.93%)	2,089.8 (−8.74%)	2,095.5 (−8.49%)
		8×10^{-4}	2,185.5 (−4.56%)	2,189.8 (−4.37%)	2,195.6 (−4.12%)
		1×10^{-3}	2,290.7 (0.03%)	2,295.1 (0.23%)	2,300.8 (0.48%)
	7.9×10^{-3}	6×10^{-4}	2,335.4 (1.99%)	2,339.8 (2.18%)	2,345.5 (2.43%)
		8×10^{-4}	2,435.6 (6.36%)	2,440.3 (6.57%)	2,446.0 (6.82%)
		1×10^{-3}	2,545.8 (11.18%)	2,550.5 (11.38%)	2,556.3 (11.63%)
	1.2×10^{-2}	6×10^{-4}	2,585.5 (12.91%)	2,590 (13.11%)	2,595.6 (13.35%)
		8×10^{-4}	2,695.6 (17.72%)	2,700.2 (17.92%)	2,705.9 (18.17%)
		1×10^{-3}	2,810.7 (22.74%)	2,815.4 (22.95%)	2,821.2 (23.20%)
2×10^{-2}	3.7×10^{-3}	6×10^{-4}	1,935.4 (−15.48%)	1,939.8 (−15.29%)	1,945.5 (−15.04%)
		8×10^{-4}	2,005.5 (−12.42%)	2,009.9 (−12.23%)	2,015.6 (−11.98%)
		1×10^{-3}	2,135.7 (−6.73%)	2,140.0 (−6.55%)	2,145.7 (−6.30%)
	7.9×10^{-3}	6×10^{-4}	2,185.4 (−4.56%)	2,189.8 (−4.37%)	2,195.5 (−4.12%)
		8×10^{-4}	2,285.3 (−0.18%)	**2,289.6(BV)**	2,295.3 (0.27%)
		1×10^{-3}	2,395.8 (4.62%)	2,399.9 (4.80%)	2,406.2 (5.08%)
	1.2×10^{-2}	6×10^{-4}	2,445.4 (6.79%)	2,449.8 (6.98%)	2,455.5 (7.23%)
		8×10^{-4}	2,545.7 (11.17%)	2,550.0 (11.36%)	2,555.6 (11.60%)
		1×10^{-3}	2,646.1 (15.56%)	2,650.2 (15.73%)	2,655.8 (15.98%)
5.2×10^{-2}	3.7×10^{-3}	6×10^{-4}	1,828.3 (−20.16%)	1,832.6 (−19.97%)	1,838.3 (−19.72%)
		8×10^{-4}	1,928.7 (−15.77%)	1,932.8 (−15.59%)	1,938.4 (15.35%)
		1×10^{-3}	2,118.8 (−7.47%)	2,122.8 (−7.30%)	2,128.6 (−7.04%)
	7.9×10^{-3}	6×10^{-4}	2,028.5 (−11.42%)	2,032.7 (−11.23%)	2,038.6 (−10.97%)
		8×10^{-4}	2,143.6 (−6.39%)	2,147.8 (−6.21%)	2,153.7 (−5.95%)
		1×10^{-3}	2,342.8 (2.31%)	2,346.9 (2.49%)	2,352.9 (2.75%)
	1.2×10^{-2}	6×10^{-4}	2,278.5 (−0.50%)	2,283.1 (−0.30%)	2,288.7 (−0.05%)
		8×10^{-4}	2,473.6 (8.02%)	2,478.3 (8.23%)	2,483.9 (8.47%)
		1×10^{-3}	2,672.8 (16.72%)	2,677.7 (16.94%)	2,683.0 (17.17%)

Note: BV denotes the base value of the mortgage insurance premium and the values in parentheses denote the changing rate of the mortgage insurance premium when the parameters are plus (minus) two standard deviations.

is projected to decrease by two standard deviations. Similarly, this table also shows that the value of mortgage insurance contracts is positively correlated with the normal volatility of the Brownian component. Consequently, in Table 3, the abnormal volatility of jump sizes, δ^2, has the largest effect of all parameters on the mortgage insurance premium. This implies that when a new home owned by the borrower is mortgaged to the lender and the insurer writes a mortgage insurance contract that promises to compensate the lender only when the borrower defaults, the insurer must consider the impact of the jump parameters on pricing the mortgage insurance contracts.

For an extreme example, according to the data on annual housing price in Wyoming from 1978 to 2003 from the Federal Housing Financial Board, there were 12 times when the annual housing price increased over two standard deviations in a year. If we define the annual housing price increasing over three standard deviations in a year as an abnormal event, the shock frequency of the abnormal events is obtained to be 0.5 (12/24). Furthermore, the lowest annual housing price return increasing over two standard deviations is 11.3 percent in 1994, while the highest annual housing price return increasing over two standard deviations is 140 percent in 1984. Hence, the abnormal volatility of jump size and the normal volatility of the Brownian component are computed as 0.334 and 0.004, respectively. Therefore, based on the data for Wyoming's annual housing price, the mortgage insurance premium is calculated as $3,623.8. Comparing to the base valuation with national average new homes prices, the mortgage insurance premium should be increased by 14.18 percent.

CONCLUSION

In the past decades, the U.S. housing prices have been relatively stable, so for the valuation of mortgage insurance contracts, the housing price process is assumed to follow traditional geometric Brownian motion. The use of Brownian motion is a reflection of normal events, but it may not work so well for abnormal shock events. Recently, U.S. housing prices have shown significant variation after the announcement of government policy changes or catastrophic events. Thus, it is necessary to develop a suitable framework for housing price process that includes jump risk.

This article first estimates parameters of the JDM by using EM gradient algorithms based on national average new home prices and previously occupied home prices for single-family mortgage from January 1986 to June 2008. The empirical results indicate that national average new home prices for single-family mortgage have jump phenomenon, whereas national average previously occupied home prices for single-family mortgage do not. To capture the jump behavior of the U.S. housing price, this article uses the jump diffusion process to derive the closed-form formula of mortgage insurance contracts. The numerical results report the mortgage insurance premium when indicated parameters increase (decrease) two standard deviations to demonstrate the abnormal volatility effect, the abnormal mean effect, the normal volatility effect, and shock frequency effect. The numerical results show that the mortgage insurance premium is an increasing function of the abnormal variance of jump size, the shock frequency of the abnormal event, and the normal volatility of the Brownian component, whereas it is a decreasing function of the mean of jump size. Compared with the base valuation, if a housing crash occurs in the future, and if this crash then causes the abnormal volatility of jump size to increase two standard deviations, while all other parameters are fixed, the mortgage insurance premium should be increased 11.36 percent. Conversely, if the abnormal volatility of jump size is projected to decrease two standard deviations, the mortgage insurance premium should be reduced 12.23 percent. Further, if the subprime mortgage crisis occurs and continues to deepen, increasing the possibility of a future U.S. housing recession, the frequency of abnormal events in the housing market is predicted to rise. When the shock frequency of the abnormal event is predicted to increase two standard deviations, the mortgage insurance premium increases 0.27 percent. Similarly, the mortgage insurance premium increases 6.57 percent when the mean of the jump size decreases two standard deviations. Therefore, the abnormal variance of jump size has the most significant effect on the mortgage insurance premium, and this implies that the insurer must consider carefully the impact of the jump parameters on pricing the mortgage insurance contracts when the collateral asset is new homes.

In addition to jump diffusion process, the positive serial correlation of U.S. housing price movements is also an especially relevant significant from the specification based on the geometric Brownian motion (see Case and Shiller, 1989). This implies that the recent declines in U.S. housing prices are likely to be followed by additional declines or at least below average price gains, in the near future. Thus, the implications of this for the pricing of mortgage insurance would be an interesting issue and the pricing framework of some studies (see Sutton, 1995; Kuo, 1996) could provide a useful starting point when addressing this issue. Next, other potential improvements and possible extensions are given. First, the correlation between housing prices and default risk of borrowers could be considered. Finally, due to the jump part, the market is incomplete and the conventional riskless hedging is difficult to obtain, so the issue of hedging with jump risk is an important topic.

APPENDIX: LOG-LIKELIHOOD FUNCTION OF JDM VIA EM GRADIENT ALGORITHM

The complete data are postulated to have complete-data likelihood function $L_C(\theta \mid \mathrm{R}, \mathrm{N}.)$ with respect to some fixed measure.

$$\begin{aligned} L_C(\theta \mid \tilde{\mathrm{R}}, \tilde{\mathrm{N}}) &= P(\tilde{\mathrm{R}}, \tilde{\mathrm{N}} \mid \theta) = P(R_1, R_2, \ldots, R_T, N_1, N_2, \ldots, N_T \mid \theta) \\ &= P(R_1, R_2, \ldots, R_T \mid N_1, N_2, \ldots, N_T, \theta) P(N_1, N_2, \ldots, N_T \mid \theta). \end{aligned} \quad \text{(A1)}$$

Since $\tilde{\mathrm{R}} = \{R_1, R_2, \ldots, R_T\}$ and $\tilde{\mathrm{N}} = \{N_1, N_2, \ldots, N_T\}$ are mutually independent, we have

$$\begin{aligned} L_C(\theta \mid \tilde{\mathrm{R}}, \tilde{\mathrm{N}}) &= \prod_{t=1}^{T} P(R_t \mid N_t = m_t, \theta) P(N_t = m_t \mid \theta) \\ &= \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\left(\sigma_H^2 + m_t\delta^2\right)}} \exp\left\{-\left[\frac{(R_t - \mu - \lambda\varphi - m_t\varphi)^2}{2(\sigma_H^2 + m_t\delta^2)}\right]\right\} \frac{\exp(-\lambda)\lambda^{m_t}}{m_t!}. \end{aligned} \quad \text{(A2)}$$

After taking the log of complete-data likelihood function, we have

$$\log L_C(\theta \mid \tilde{\mathrm{R}}, \tilde{\mathrm{N}}) = \sum_{t=1}^{T} \Bigg[-\lambda + m_t \log \lambda - \log(m_t!) - \frac{1}{2} \log\left(2\pi\left(\sigma_H^2 + m_t\delta^2\right)\right) - \frac{(R_t - \mu - \lambda\varphi - m_t\varphi)^2}{2\left(\sigma_H^2 + m_t\delta^2\right)} \Bigg]. \tag{A3}$$

Hence, in the E-step of the EM algorithm, the conditional expectation of the complete-data likelihood function

$$\begin{aligned} Q(\theta \mid \theta_{\mathrm{n}}) &= E(\log L_C(\theta \mid \tilde{\mathrm{R}}, \tilde{\mathrm{N}}) \mid \tilde{\mathrm{R}}, \theta_{\mathrm{n}}) = \sum_{t=1}^{T} E(\log L_C(\theta \mid R_t, N_t) \mid R_t, \theta_{\mathrm{n}}) \\ &= \sum_{t=1}^{T} \sum_{m_t=0}^{\infty} \Bigg[-\lambda + m_t \log \lambda - \log(m_t!) - \frac{1}{2} \log\left(2\pi\left(\sigma_H^2 + m_t\delta^2\right)\right) \\ &\quad - \frac{(R_t - \mu - \lambda\varphi - m_t\varphi)^2}{2\left(\sigma_H^2 + m_t\delta^2\right)} \Bigg] P(N_t = m_t \mid R_t, \theta_{\mathrm{n}}) \end{aligned} \tag{A4}$$

is computed, where $\theta_{\mathrm{n}} = (\mu_n, \sigma_{Hn}^2, \varphi_n, \delta_n^2, \lambda_n)$ is the current estimated value of θ.

Furthermore, $P(N_t = m_t \mid R_t, \theta_{\mathrm{n}})$ can be computed as:

$$P(N_t = m_t \mid R_t, \theta_{\mathrm{n}}) = \frac{L_C(\theta_{\mathrm{n}} \mid R_t, N_t = m_t)}{\sum_{m_t=0}^{\infty} L_C(\theta_{\mathrm{n}} \mid R_t, N_t = m_t)}. \tag{A5}$$

Next, a vector $\tilde{\theta} = (\tilde{\mu}, \tilde{\sigma}_H^2, \tilde{\varphi}, \tilde{\delta}^2, \tilde{\lambda})$ that maximizes $Q(\theta \mid \theta_{\mathrm{n}})$ with respect to θ is found. This is called the M-step. Replacing θ_{n} by $\tilde{\theta}$ and repeating the E- and M-steps produces a sequence of values of θ_{n} that converges to an MLE $\tilde{\theta}$.

Under the EM gradient algorithm, the current parameter column vector θ_{n} is updated by

$$\theta_{\mathrm{n}+1} = \theta_{\mathrm{n}} - \left(d^{20} Q(\theta_{\mathrm{n}} \mid \theta_{\mathrm{n}})\right)^{-1} d^{10} Q(\theta_{\mathrm{n}} \mid \theta_{\mathrm{n}}), \tag{A6}$$

where

$$d^{10}Q(\theta\mid\theta_{\mathrm{n}})=\begin{bmatrix}\dfrac{\partial Q(\theta\mid\theta_{\mathrm{n}})}{\partial\mu}\\ \dfrac{\partial Q(\theta\mid\theta_{\mathrm{n}})}{\partial\sigma_H^2}\\ \dfrac{\partial Q(\theta\mid\theta_{\mathrm{n}})}{\partial\varphi}\\ \dfrac{\partial Q(\theta\mid\theta_{\mathrm{n}})}{\partial\delta^2}\\ \dfrac{\partial Q(\theta\mid\theta_{\mathrm{n}})}{\partial\lambda}\end{bmatrix}$$

$$=\begin{bmatrix}\displaystyle\sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[\frac{(R_t-\mu-\lambda\varphi-m_t\varphi)}{\left(\sigma_H^2+m_t\delta^2\right)}\right]P(N_t=m_t\mid\tilde{\mathrm{R}},\theta_{\mathrm{n}})\\ \displaystyle\sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[-\frac{1}{2}\frac{1}{\left(\sigma_H^2+m_t\delta^2\right)}+\frac{(R_t-\mu-\lambda\varphi-m_t\varphi)^2}{2\left(\sigma_H^2+m_t\delta^2\right)^2}\right]P(N_t=m_t\mid\tilde{\mathrm{R}},\theta_{\mathrm{n}})\\ \displaystyle\sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[\frac{(R_t-\mu-\lambda\varphi-m_t\varphi)(\lambda+m_t)}{\left(\sigma_H^2+m_t\delta^2\right)}\right]P(N_t=m_t\mid\tilde{\mathrm{R}},\theta_{\mathrm{n}})\\ \displaystyle\sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[-\frac{1}{2}\frac{m_t}{\left(\sigma_H^2+m_t\delta^2\right)}+\frac{(R_t-\mu-\lambda\varphi-m_t\varphi)^2m_t}{2\left(\sigma_H^2+m_t\delta^2\right)^2}\right]P(N_t=m_t\mid\tilde{\mathrm{R}},\theta_{\mathrm{n}})\\ \displaystyle\sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[-1+\frac{m_t}{\lambda}\right]P(N_t=m_t\mid\tilde{\mathrm{R}},\theta_{\mathrm{n}})\end{bmatrix}$$

$$d^{20}Q(\theta\mid\theta_{\mathrm{n}})=\begin{bmatrix}\dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\mu^2} & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\mu\partial\sigma_H^2} & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\mu\partial\varphi} & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\mu\partial\delta^2} & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\mu\partial\lambda}\\ & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\sigma_H^4} & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\sigma_H^2\partial\varphi} & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\sigma_H^2\partial\delta^2} & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\sigma_H^2\partial\lambda}\\ & & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\varphi^2} & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\varphi\partial\delta^2} & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\varphi\partial\lambda}\\ & & & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\delta^4} & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\delta^2\partial\lambda}\\ & & & & \dfrac{\partial^2Q(\theta\mid\theta_{\mathrm{n}})}{\partial\lambda^2}\end{bmatrix}$$

where

$$\frac{\partial^2 Q(\theta \mid \theta_n)}{\partial \mu^2} = \sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[\frac{-1}{(\sigma_H^2 + m_t\delta^2)}\right]P(N_t = m_t \mid \tilde{R}, \theta_n),$$

$$\frac{\partial^2 Q(\theta \mid \theta_n)}{\partial \mu \partial \sigma_H^2} = \sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[-\frac{(R_t - \mu - \lambda\varphi - m_t\varphi)}{(\sigma_H^2 + m_t\delta^2)^2}\right]P(N_t = m_t \mid \tilde{R}, \theta_n),$$

$$\frac{\partial^2 Q(\theta \mid \theta_n)}{\partial \mu \partial \varphi} = \sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[\frac{-(\lambda + m_t)}{(\sigma_H^2 + m_t\delta^2)}\right]P(N_t = m_t \mid \tilde{R}, \theta_n),$$

$$\frac{\partial^2 Q(\theta \mid \theta_n)}{\partial \mu \partial \delta^2} = \sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[\frac{-m_t(R_t - \mu - \lambda\varphi - m_t\varphi)}{(\sigma_H^2 + m_t\delta^2)^2}\right]P(N_t = m_t \mid \tilde{R}, \theta_n), \quad \frac{\partial^2 Q(\theta \mid \theta_n)}{\partial \mu \partial \lambda} = 0,$$

$$\frac{\partial^2 Q(\theta \mid \theta_n)}{\partial \sigma_H^4} = \sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[\frac{\frac{1}{2}(\sigma_H^2 + m_t\delta^2) - (R_t - \mu - \lambda\varphi - m_t\varphi)^2}{(\sigma_H^2 + m_t\delta^2)^3}\right]P(N_t = m_t \mid \tilde{R}, \theta_n),$$

$$\frac{\partial^2 Q(\theta \mid \theta_n)}{\partial \sigma_H^2 \partial \varphi} = \sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[-\frac{(\lambda + m_t)(R_t - \mu - \lambda\varphi - m_t\varphi)}{(\sigma_H^2 + m_t\delta^2)^2}\right]P(N_t = m_t \mid \tilde{R}, \theta_n),$$

$$\frac{\partial^2 Q(\theta \mid \theta_n)}{\partial \sigma_H^2 \partial \delta^2} = \sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[\frac{m_t\left(\frac{1}{2}(\sigma_H^2 + m_t\delta^2) - (R_t - \mu - \lambda\,\varphi - m_t\,\varphi)^2\right)}{(\sigma_H^2 + m_t\delta^2)^3}\right] \times P(N_t = m_t \mid \tilde{R}, \theta_n), \quad \frac{\partial^2 Q(\theta \mid \theta_n)}{\partial \sigma_H^2 \partial \lambda} = 0,$$

$$\frac{\partial^2 Q(\theta \mid \theta_n)}{\partial \varphi^2} = \sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[\frac{-(\lambda + m_t)^2}{(\sigma_H^2 + m_t\delta^2)}\right]P(N_t = m_t \mid \tilde{R}, \theta_n),$$

$$\frac{\partial^2 Q(\theta \mid \theta_n)}{\partial\varphi\partial\delta^2} = \sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[\frac{-(\lambda + m_t)\, m_t(R_t - \mu - \lambda\,\varphi - m_t\,\varphi)}{\left(\sigma_H^2 + m_t\delta^2\right)^2}\right] P(N_t = m_t \mid \tilde{R}, \theta_n),$$

$$\frac{\partial^2 Q(\theta \mid \theta_n)}{\partial\delta^4} = \sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[\frac{m_t^2\left(\frac{1}{2}(\sigma_H^2 + m_t\delta^2) - (R_t - \mu - m_t\,\varphi)^2\right)}{\left(\sigma_H^2 + m_t\delta^2\right)^3}\right] P(N_t = m_t \mid \tilde{R}, \theta_n),$$

$$\frac{\partial^2 Q(\theta \mid \theta_n)}{\partial\varphi\partial\lambda} = 0, \quad \frac{\partial^2 Q(\theta \mid \theta_n)}{\partial\delta^2\partial\lambda} = 0, \quad \frac{\partial^2 Q(\theta \mid \theta_n)}{\partial\lambda^2} = \sum_{m_t=0}^{\infty}\sum_{t=1}^{T}\left[-\frac{m_t}{\lambda^2}\right] P(N_t = m_t \mid \tilde{R}, \theta_n).$$

REFERENCES

Abraham, J. M., and P. H. Hendershott, 1996, Bubbles in Metropolitan Housing Markets, *Journal of Housing Research*, 7(2): 191-207.

Andersen, T., F. X. Diebold, and T. Bollerslev, 2007, Roughing It Up: Including Jump Components in the Measurement, Modeling, and Forecasting of Return Volatility, *Review of Economics and Statistics*, 89: 701-720.

Ballotta, L., 2005, A L'evy Process-Based Framework for the Fair Valuation of Participating Life Insurance Contracts, *Insurance: Mathematics and Economics*, 37(2): 173-196.

Bardhan, A., R. Karapandža, and B. Urošević, 2006, Valuing Mortgage Insurance Contracts in Emerging Market Economies, *Journal of Real Estate Finance and Economics*, 32: 9-20.

Barndor-Nielsen, O., and N. Shephard, 2006, Econometrics of Testing for Jumps in Financial Economics Using Bipower Variation, *Journal of Financial Econometrics*, 4: 1-30.

Bin, O., J. B. Kruse, and C. E. Landry, 2008, Flood Hazards, Insurance Rates, and Amenities: Evidence From the Coastal Housing Market, *Journal of Risk and Insurance*, 75: 63-82.

Borio, C., and P. Mcguire, 2004, Twin Peaks in Equity and Housing Prices? *BIS Quarterly Review*, (March): 79-93.

Canner, G. B., and W. Passmore, 1994, Private Mortgage Insurance, *Federal Reserve Bulletin*, 9: 883-899.

Campbell, T. S., and K. Dietrich, 1983, The Determinants of Default on Insured Conventional Residential Mortgage Loans, *Journal of Finance*, 38: 1569-1581.

Case, K. E., and R. J. Shiller, 1989, The Efficiency of the Market for Single-Family Homes, *American Economic Review*, (March): 125-137.

Caselli, S., S. Gatti, and F. Querci, 2008, The Sensitivity of the Loss Given Default Rate to Systematic Risk: New Empirical Evidence on Bank Loans, *Journal of Financial Services Research*, 34: 1-34.

Chernov, M., A. R. Gallant, E. Ghysels, and G. Tauchen, 2003, Alternative Models for Stock Price Dynamics, *Journal of Econometrics*, 116: 225-257.

Dempster, A. P., N. M. Laird, and D. B. Rubin, 1977, Maximum Likelihood From Incomplete Data via the EM Algorithm, *Journal of the Royal Statistical Society: Series B*, 39: 1-38.

Deng, Y., J. M. Quigley, and R. Van Order, 2000, Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options, *Econometrica*, 68: 275-307.

Dennis, B., C. Kuo, and T. Yang, 1997, Rationales of Mortgage Insurance Premium Structures, *Journal of Real Estate Research*, 14(3): 359-378.

Durham, G. B., and A. R. Gallant, 2002, Numerical Techniques for Maximum Likelihood Estimation of Continuous-Time Diffusion Processes, *Journal of Business and Economic Statistics*, 20: 297-316.

Englund, P., and Y. M. Ioannides, 1997, House Price Dynamics: An International Empirical Perspective, *Journal of Housing Economics*, 6(2): 119-136.

Eraker, B., M. Johannes, and N. G. Polson, 2003, The Impact of Jumps in Returns and Volatility, *Journal of Finance*, 53: 1269-1300.

Gerber, H. U., and E. S. W. Shiu, 1994, Option Pricing by Esscher Transforms (With Discussion), *Transactions of the Society of Actuaries*, 46: 99-140 (Discussion, 141-191).

Harris, J. C., 1989, The Effect of Real Rates of Interest on Housing Prices, *Journal of Real Estate Finance and Economics*, 2(1): 47-60.

Hendershott, P., and R. Van Order, 1987, Pricing Mortgages: Interpretation of the Models and Results, *Journal of Financial Services Research*, 1(1): 19-55.

Huang, X., and G. Tauchen, 2005, The Relative Contribution of Jumps to Total Price Variance, *Journal of Financial Econometrics*, 3(4): 456-499.

Kau, J., and D. Keenan, 1995, An Overview of the Option-Theoretic Pricing of Mortgages, *Journal of Housing Research*, 6(2): 217-244.

Kau, J., and D. Keenan, 1996, An Option-Theortic Model of Catastrophes Applied to Mortgage Insurance, *Journal of Risk and Insurance*, 63(4): 639-656.

Kau, J., and D. Keenan, 1999, Catastrophic Default and Credit Risk for Lending Institutions, *Journal of Financial Services Research*, 15(2): 87-102.

Kau, J., D. Keenan, and W. Muller, 1993, An Option-Based Pricing Model of Private Mortgage Insurance, *Journal of Risk and Insurance*, 60(2): 288-299.

Kau, J., D. Keenan, W. Muller, and J. Epperson, 1992, A Generalized Valuation Model for Fixed-Rate Residential Mortgages, *Journal of Money, Credit and Banking*, 24: 280-299.

Kau, J. B., D. C. Keenan, W. J. Muller, and J. E. Epperson, 1995, The Valuation at Origination of Fixed-Rate Mortgages With Default and Prepayment, *Journal of Real Estate Finance and Economics*, 11: 3-36.

Kostas, T., and H. Zhu, 2004, What Drives Housing Price Dynamics: Cross-Country Evidence, *BIS Quarterly Review*, (March): 65-78.

Kuo, C. L., 1996, Housing Price Dynamics and the Valuation of Mortgage Default Options, *Journal of Housing Economics*, 5: 18-40.

Kvarnstrom, K. M., 2005, Estimation of the Diffusion Coefficient in a Mixture Model, *Kwantitatieve Methoden*, 72: 1-23.

Lambrecht, B., W. Perraudin, and S. Satchell, 2003, Mortgage Default and Possession Under Recourse: A Competing Hazards Approach, *Journal of Money, Credit, and Banking*, 35(3): 425-442.

Lange, K., 1995, A Gradient Algorithm Locally Equivalent to the EM Algorithm, *Journal of the Royal Statistical Society: Series B. Methodological*, 57(2): 425-437.

Mizrach, B., 2008, Jump and Co-Jump Risk in Subprime Home Equity Derivatives, Working Paper, Department of Economics, Rutgers University.

Nityasuddhi, D., and D. Bohning, 2003, Asymptotic Properties of the EM Algorithm Estimate for Normal Mixture Models With Component Specific Variances, *Computational Statistics and Data Analysis*, 41: 591-601.

Pan, J., 2002, The Jump-Risk Premia Implicit in Options: Evidence From an Integrated Time-Series Study, *Journal of Financial Economics*, 63: 3-50.

Quigley, J. M., and R. Van Order, 1990, Efficiency in the Mortgage Market: The Borrower's Perspective, *Journal of the AREUEA*, 18: 237-252.

Quigley, J. M., and R. Van Order, 1995, Explicit Tests of Contingent Claims Models of Mortgage Default, *The Journal of Real Estate Finance and Economics*, 11: 99-117.

Schloemer, E., W. Li, K. Ernst, and K. Keist, 2006, Losing Ground: Foreclosures in the Subprime Markets and Their Cost to Homeowners, Working Paper, Center for Responsible Lending.

Schwartz, E. S., and W. N. Torous, 1989, Prepayment and the Valuation of Mortgage-Backed Securities, *Journal of Finance*, 44: 375-392.

Schwartz, E. S., and W. N. Torous, 1993, Mortgage Prepayment and Default Decisions: A Poisson Regression Approach, *Journal of the AREUEA*, 21: 431-449.

Sutton, G. D., 2002, Explaining Changes in House Prices, *BIS Quarterly Review*, (September): 46-55.

Sutton, G. D., 1995, Price Inertia and the Valuation of Real Estate Options, Unpublished PhD Dissertation, Yale University.

Tanner, M. A., 1996, *Tools for Statistical Inference* (Heidelberg: Springer-Verlag).

Zeng, D., and D. Cai, 2005, Asymptotic Results for Maximum Likelihood Estimators in Joint Analysis of Repeated Measurements and Survival Time, *Annals of Statistics*, 33: 2132-2163.