



Purchasing Power Parity (購買力平價理論)

The Law of One Price(單一訂價法則)

- in a PCM(Perfect Capital Market , 完全資本市場) setting, goods will sell for the same price in two markets, taking into account the exchange rate

$$P_{d, \text{iphone}} = S \times P_{f, \text{iphone}}$$

Absolute purchasing power parity(絕對購買力平價)

$$P_d = S \times P_f \quad -(1)$$

Relative purchasing power (相對購買力平價理論)

(1) the classic statement of relative PPP

Absolute PPP is violated

$$Pd_{t+1} = K \times S_{t+1} \times Pf_{t+1} \quad -(4.3a)$$

$$Pd_t = K \times S_t \times Pf_t \quad -(4.3b)$$

(4.3a) divided by (4.3b)

$$(Pd_{t+1} / Pd_t) = (S_{t+1} / S_t) \times (Pf_{t+1} / Pf_t)$$

$$\underline{\dot{S} = \dot{P}_d - \dot{P}_f}$$

=> 匯率變動率 = 本國通膨率 - 外國通膨率

$$\dot{S} = (S_{t+1} - S_t) / S_t \quad (\text{匯率變動率})$$

$$\dot{P}_d = (P_{d,t+1} - P_{d,t}) / P_{d,t} \quad (\text{本國通膨率})$$

$$\dot{P}_f = (P_{f,t+1} - P_{f,t}) / P_{f,t} \quad (\text{外國通膨率})$$

(2) the level of the exchange rate that satisfies PPP

(4.3a) divided by (4.3b) $\Rightarrow S_{t+1}/S_t = (P_{d,t+1}/P_{f,t+1}) / (P_{d,t}/P_{f,t})$

$$\diamond \quad S_{\text{PPP}, t+1} = S_t \times (P_{d,t+1}/P_{d,t}) / (P_{f,t+1}/P_{f,t})$$

Ex: Nominal exchange rate in the base period was \$1.50; the prices of U.S. goods had risen by 8%; the prices of U.K. goods had risen by 4%. What is the PPP spot rate?

$$\begin{aligned} S_{\text{PPP}, t+1} &= S_t \times (P_{US,t+1}/P_{US,t}) / (P_{UK,t+1}/P_{UK,t}) \\ &= 1.5 \times (1.08 / 1.04) = 1.5577 \end{aligned}$$

\Rightarrow (A) nominal exchange rate $> S_{\text{PPP}, t+1}$ (overshooting)

(B) nominal exchange rate $< S_{\text{PPP}, t+1}$ (undershooting)

(C) if nominal exchange rate $= S_{\text{PPP}, t+1}$

(international price competitiveness are unchanged relative to the base period.)

(3) real exchange rates(實質匯率)

$$\text{real exchange rate} = \frac{\text{the price of foreign goods}}{\text{the price of domestic goods}} = \frac{S.P_f}{P_d}$$

(a) the real exchange rate measures the international price competitiveness of domestic versus foreign products.

(衡量國際價格競爭力)

(b) The real exchange rate is unchanged when the exchange rate changes by the same percentage as the relative price of the two countries.(兩國物價同比例變化，實質匯率不變)

(c) The real exchange rate is constant in periods when PPP holds.(ppp成立時，實質匯率為一固定數值)

How to calculate the real exchange rate

$$\text{Spot(Real ,t)} = \frac{\text{Spot(Nominal ,t)}}{\text{Spot(PPP ,t)}} \quad (4.9)$$

Ex: $S_t = \$1.80/\text{£}$, PPP spot rate = $\$1.50/\text{£}$

$$\rightarrow \text{Spot (Real)} = \text{Spot (Nominal)} / \text{Spot (PPP)} = 1.80/1.50 \\ = 1.20 \text{ or } 120 \text{ (the real exchange rate)}$$

\Rightarrow **£ is overvalued (高估)** ($\because \$1.80/\text{£} > \text{Spot(PPP ,t)} = \$1.50/\text{£}$)
\$ is undervalued (低估)

or 1.0 British good can be exchanged for 1.2 U.S. goods

\Rightarrow sellers of British goods have “lost competitiveness” on international markets – for every \$1.80 worth of international money, a buyer could purchase either 1.0 British good for 1.2 U.S. goods.

2.relaxing the perfect capital maket assumptions

transaction costs (交易成本)

- ◆ a deviation from parity $>$ transaction costs
=> will arbitrage
 - ◆ neutral band (中立區間) –
deviations from parity $<$ transaction cost
=> no arbitrage
- ∴ even when transaction costs are present, we can continue to think of PPP as a useful benchmark when adjusted by an appropriate measure of the neutral band
(即使有交易成本，可以ppp做為參考基準值)

Taxes (租稅)

- taxes have an effect similar to transaction costs; a tax of this type will simply widen the neutral band
(租稅會加大中立區間)

Uncertainty(不確定性)

- risk-averse arbitrageurs(避險套利者) will seek a greater profit to compensate for these risks, thus leading to a wider neutral band around PPP exchange rates .
(不確定性會使套利者追求更大利潤來補償風險，因此加大ppp線中立區間)

3. empirical evidence on prices and exchange rates –

Evidence on the law of one price

the Big Mac Index (the Economist) <http://www.economist.com/content/big-mac-index>

Country	Local Price	Dollar Exchange	Dollar Price	Dollar PPP	Implied Dollar Valuation	Dollar price /US price
Taiwan	79	31.49	2.51	16.49	-47.63	0.524008351
Thailand	99	32.61	3.04	20.67	-36.61	0.634655532
Turkey	9.25	2.33	3.96	1.93	-17.24	0.826722338
UAE	13	3.67	3.54	2.71	-26.11	0.739039666
Ukraine	19	15.82	1.2	3.97	-74.93	0.250521921
U.S.	4.79	1	4.79	1	0	1
Uruguay	113	24.43	4.63	23.59	-3.42	0.966597077
Venezuela	132	52.1	2.53	27.56	-47.11	0.528183716
Vietnam	60000	21380	2.81	12526.1	-41.41	0.586638831
Austria	3.39	0.86	3.93	0.71	-18	0.82045929
Belgium	3.7	0.86	4.29	0.77	-10.5	0.895615866
Estonia	2.9	0.86	3.36	0.61	-29.85	0.701461378
Finland	4.1	0.86	4.75	0.86	-0.83	0.991649269
France	3.9	0.86	4.52	0.81	-5.66	0.943632568
Germany	3.67	0.86	4.25	0.77	-11.23	0.887265136
Greece	3.05	0.86	3.53	0.64	-26.22	0.736051083

Country	Dollar/national exchange rat	\$1 in terms of the Big Mac Index expressed in national currency	Price of Big Mac	Currency is undervalued / overvalued by %
South Africa	16.67	5.43	\$ 1,86	-67.44
Russia	70.59	23.64	\$ 1,91	-66.51
Turkey	6.86	2.45	\$ 2,04	-64.28
Ukraine	27.13	10.33	\$ 2,17	-61.91
Mexico	22.44	8.76	\$ 2,23	-60.97
Romania	4.26	1.73	\$ 2,32	-59.30
Azerbaijan	1.70	0.69	\$ 2,32	-59.28
Malaysia	4.27	1.75	\$ 2,34	-58.98
Indonesia	14435	5954.47	\$ 2,36	-58.75
Taiwan	29.46	12.61	\$ 2,44	-57.19
India	75.20	33.27	\$ 2,53	-55.75
Egypt	15.95	7.36	\$ 2,63	-53.88
Hong Kong	7.75	3.59	\$ 2,64	-53.68
Moldova	17.08	8.23	\$ 2,75	-51.79
Poland	3.94	1.93	\$ 2,79	-51.15
Vietnam	23180	11558.67	\$ 2,85	-50.14
Oman	0.39	0.19	\$ 2,86	-49.96
Philippines	49.43	24.87	\$ 2,87	-49.69
Hungary	311.38	157.62	\$ 2,89	-49.38
China	7.01	3.80	\$ 3,10	-45.74
Jordan	0.71	0.40	\$ 3,24	-43.19
Guatemala	7.69	4.38	\$ 3,25	-43.09
Colombia	3617	2084.06	\$ 3,29	-42.38
Pakistan	166.50	96.32	\$ 3,30	-42.15
Croatia	6.63	3.85	\$ 3,32	-41.86
Peru	3.50	2.08	\$ 3,40	-40.41
Chile	787.70	479.86	\$ 3,48	-39.08
Nicaragua	34.38	21.02	\$ 3,49	-38.86

Country	Dollar/national currency exchange rat	\$1 in terms of the Big Mac Index expressed in national currency	Price of Big Mac	Currency is undervalued / overvalued by %	
Argentina	71.24	43.78	\$ 3,51		-38.54
Honduras	24.69	15.24	\$ 3,52		-38.29
Qatar	3.64	2.28	\$ 3,57		-37.47
Japan	107.28	68.30	\$ 3,64		-36.33
Sri Lanka	185.85	119.09	\$ 3,66		-35.92
Bahrain	0.38	0.25	\$ 3,71		-34.97
Saudi Arabia	3.75	2.45	\$ 3,73		-34.64
Kuwait	0.31	0.20	\$ 3,74		-34.57
South Korea	1200.95	788.09	\$ 3,75		-34.38
Czech Republic	23.41	15.59	\$ 3,80		-33.43
Brazil	5.34	3.66	\$ 3,91		-31.46
United Arab Emirates	3.67	2.58	\$ 4,02		-29.67
Costa Rica	581.83	411.56	\$ 4,04		-29.26
Thailand	31.39	22.42	\$ 4,08		-28.58
Singapore	1.39	1.03	\$ 4,25		-25.54
Britain	0.79	0.59	\$ 4,28		-25.09
Uruguay	43.68	33.10	\$ 4,33		-24.21
New Zealand	1.52	1.16	\$ 4,35		-23.85
Australia	1.43	1.15	\$ 4,58		-19.82
Denmark	6.55	5.25	\$ 4,58		-19.78
Euro area	0.88	0.74	\$ 4,79		-16.18
Israel	3.44	2.98	\$ 4,95		-13.37
Canada	1.36	1.20	\$ 5,08		-11.09
Norway	9.37	9.11	\$ 5,55		-2.82
United States	1	1	\$ 5,71		0
Sweden	9.14	9.21	\$ 5,76		0.80
Lebanon	1512	1576.18	\$ 5,95		4.25
Switzerland	0.94	1.14	\$ 6,91		20.94



Country	2000 — 2021	Under/over valued, %
Guatemala	Quetzal	-40.6
Peru	Sol	-42.2
Mexico	Peso	-43.7
Jordan	Dinar	-46.8
Vietnam	Dong	-47.0
Oman	Rial	-47.1
Hungary	Forint	-47.9
Moldova	Leu	-48.8
Philippines	Peso	-50.1
Egypt	Pound	-52.0
Hong Kong	HK\$	-52.2
Taiwan	NT\$	-54.6
India	Rupee	-54.9

Choose a base currency

US dollar

Show index at

Jul 2021

for GDP per person

Raw index

GDP-adjusted

The Taiwanese dollar is **54.6% undervalued** against the US dollar
July 2021



A Big Mac costs NT\$72 in Taiwan and US\$5.65 in the United States. The implied exchange rate is 12.74. The difference between this and the actual exchange rate, 28.05, suggests the Taiwanese dollar is **54.6% undervalued**

2000-2021

GDP Per Capita Ranking in 2017(IMF)

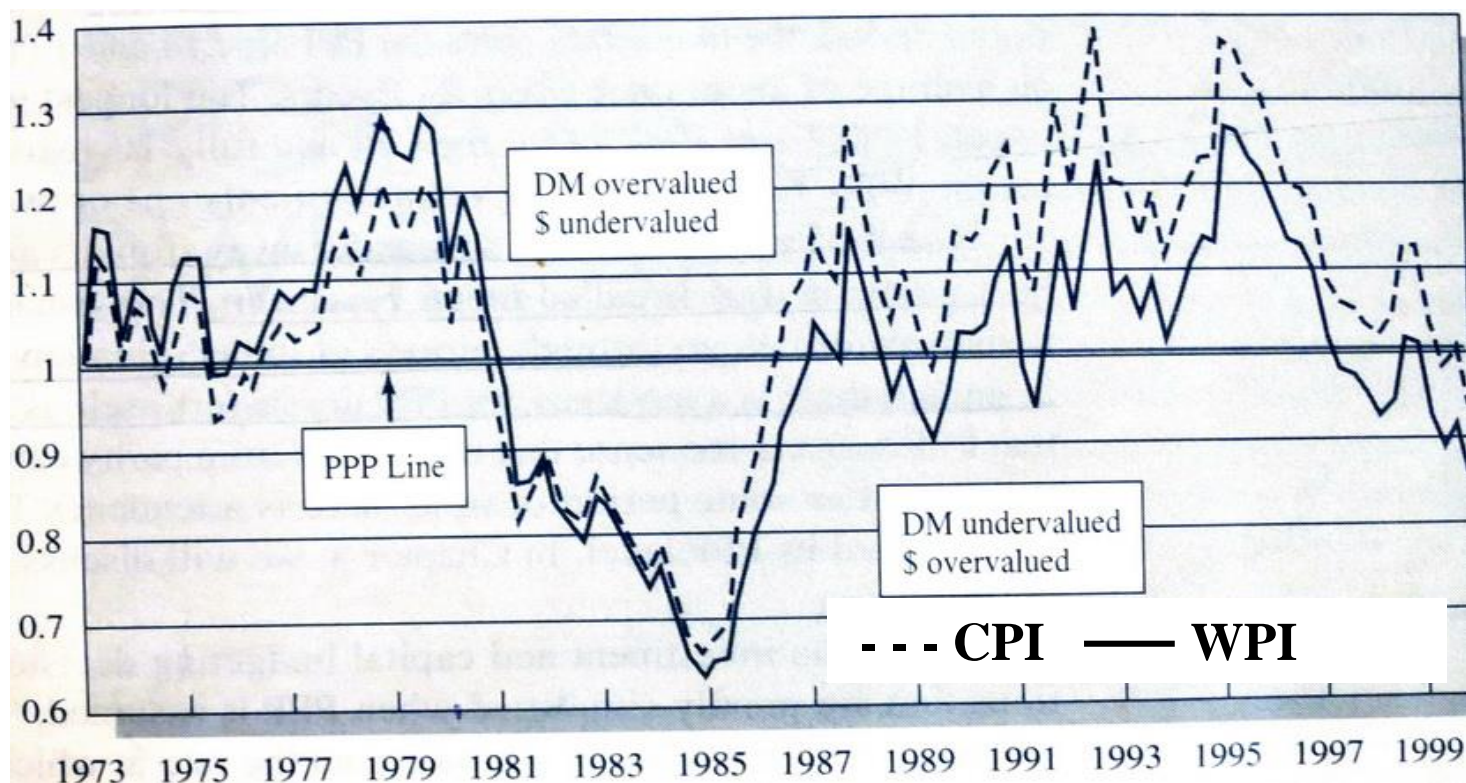
	人均gdp (排名) GDP Per Capita (ranking)	GDP (PPP) Per Capita (ranking)
新加坡(Singapore)	57,713 (8)	90,531 (3)
美國(U.S.)	59,501 (7)	59,495 (11)
香港(HK)	46,109 (15)	64,533 (6)
德國(Germany)	44,550 (17)	50,206 (17)
英國(U.K.)	39,735 (22)	43,620 (26)
法國(France)	39,869 (21)	43,550 (27)
日本(Japan)	38,440 (23)	42,659 (28)
南韓(South Korea)	29,891 (27)	39,387 (30)
台灣(Taiwan)	24,577 (33)	49,827 (19)
中國(China)	8,643 (72)	16,624 (79)

relative ppp: evidence on recent quarterly data

(1) PPP calculations(Appendix A)

$$S_{PPP,t+n} = S_{\$/DM,0} \frac{P_{US,t+n} / P_{G,t+n}}{P_{US,0} / P_{G,0}}$$

Fig. 4.4 PPP:Germany and the U.S., 1973-1999



APPENDIX A Purchasing Power Parity Calculations
United States and Germany

ln (S_{PPP})

<i>①</i> Date: Year and Quarter	<i>②</i> \$/DM Spot, End of Period	<i>③</i> Germany CPI	<i>④</i> U.S. CPI	<i>⑤</i> CPI (US)/ CPI (G)	<i>⑥</i> PPP Spot	<i>⑦</i> Ratio Actual/ PPP	<i>⑧</i> Percent Deviation from PPP
1973Q1	0.3523	46.82	28.20	0.6023	0.3523	1.0000	0.0000
1973Q2	0.4124	47.73	28.82	0.6038	0.3532	1.1676	0.1549
1973Q3	0.4140	48.15	29.45	0.6116	0.3578	1.1572	0.1460
1973Q4	0.3700	49.10	30.14	0.6138	0.3591	1.0304	0.0299
1974Q1	0.3964	50.27	30.99	0.6165	0.3606	1.0992	0.0946
1974Q2	0.3914	51.08	31.86	0.6237	0.3648	1.0728	0.0703
1974Q3	0.3770	51.55	32.83	0.6369	0.3725	1.0120	0.0119
1974Q4	0.4150	52.25	33.79	0.6467	0.3783	1.0972	0.0927
1975Q1	0.4264	53.25	34.40	0.6460	0.3779	1.1286	0.1209
1975Q2	0.4247	54.26	34.94	0.6439	0.3766	1.1275	0.1200
1975Q3	0.3757	54.68	35.69	0.6527	0.3818	0.9842	-0.0160
1975Q4	0.3813	55.15	36.26	0.6575	0.3846	0.9916	-0.0084
1976Q1	0.3940	56.03	36.61	0.6534	0.3822	1.0308	0.0304
1976Q2	0.3885	56.67	37.06	0.6540	0.3825	1.0156	0.0155
1976Q3	0.4104	56.80	37.65	0.6629	0.3877	1.0586	0.0569
1976Q4	0.4233	57.18	38.08	0.6660	0.3895	1.0866	0.0831
1977Q1	0.4186	58.15	38.75	0.6664	0.3898	1.0740	0.0714
1977Q2	0.4277	58.77	39.58	0.6735	0.3939	1.0858	0.0823
1977Q3	0.4334	58.94	40.16	0.6814	0.3985	1.0874	0.0838
1977Q4	0.4751	59.19	40.60	0.6859	0.4012	1.1841	0.1690
1978Q1	0.4943	59.91	41.29	0.6892	0.4031	1.2262	0.2039
1978Q2	0.4819	60.44	42.36	0.7009	0.4099	1.1754	0.1616
1978Q3	0.5158	60.40	43.36	0.7179	0.4199	1.2285	0.2058
1978Q4	0.5475					1.2811	0.2477
1979Q1	0.5350					1.2449	0.2191
1979Q2	0.5410					1.2327	0.2092
1979Q3	0.5730					1.2817	0.2482
1979Q4	0.5775					1.2642	0.2345
1980Q1	0.5150	65.08	51.81	0.7961	0.4656	1.1059	0.1007
1980Q2	0.5688	66.14	53.68	0.8116	0.4747	1.1981	0.1807
1980Q3	0.5521	66.54	54.69	0.8219	0.4807	1.1484	0.1384
1980Q4	0.5105	67.16	56.12	0.8356	0.4888	1.0444	0.0434

$$S_{PPP, t+n} = S_{\$/DM, 0} \frac{P_{US, t+n} / P_{G, t+n}}{P_{US, 0} / P_{G, 0}}$$

(2)a graph of the real exchange rate (Fig 4.4, based on eq.(4.9), use CPI & WPI)

$$\text{Spot(Real ,t)} = \frac{\text{Spot(Nominal ,t)}}{\text{Spot(PPP ,t)}} \quad (4.9)$$

- ◆ 1979.3 – 28% real DM appreciation(**the German exporters lost competition**，實質德馬克高估，德出口商失競爭力)
- ◆ 1984.4 - 38% real US\$ appreciation (**the U.S. exporters lost competition**，實質美元高估，美出口商失競爭力)

- (a) But there appears to be a tendency for real rate to return to PPP, in the sense that each series tends to cross the PPP line every few years.

(圖中實質匯率線每隔幾年會與ppp線相交，顯示實質匯率有向ppp理論值修正的趨勢)

- (b) In these examples, counting the initial period, the two series cross the PPP line 14 and 12 times in 27 years, an average of about once every 2-3 years.

(樣本期間之27年內，兩序列曲線與PPP線相交分別為14與12次，即每2-3年會交叉一次)

(C) using the CPI(消費者物價指數)

- the average deviation from parity in any quarter was a 0.02%
=> not significantly different from zero.

(偏離PPP平均每季為 0.02% => 不顯著異於零)

- the average length of time required to cross the PPP line
was less 2 years (平均兩年內會與PPP線相交)

(d) using the WPI (躉售物價指數)

- the average deviation from parity in any quarter was a 4.1%
=> significantly different from zero.

(偏離PPP平均每季為 4.1% => 顯著異於零)

**** These findings suggest that PPP held on average over the period using the CPI, but not using the WPI.**

(長期下以CPI計算時，其結果PPP會成立，但以WPI計算結果則不支持PPP)

(3) regression test

- ◆ The empirical study of “PPP” can use the following equation

$$\dot{S}_t = \alpha + \beta (\dot{P}_{d,t} - \dot{P}_{f,t}) + \varepsilon_t$$

If PPP is true, null Hypothesis $H_0: \beta=0$

$$\dot{S} = d \log S = dS/S$$

$$\dot{P}_d = d \log P_d = d P_d / P_d$$

$$\dot{P}_f = d \log P_f = d P_f / P_f$$

the OLS(Ordinary least Square, 最小平方法) result of PPP

the OLS result of PPP (以OLS估計PPP之範例)

Oct. 1991- Jun. 2012 , the data of Canada and the U. S.

the model as the following:

$$\ln S_t = \alpha + \beta (\ln P_{can, t} - \ln P_{US, t}) + \varepsilon_t$$

$$d \ln S_t / d S_t = 1 / S_t \Rightarrow d \ln S_t = d S_t / S_t$$

$$d \ln P_{can, t} / d P_{can, t} = 1 / P_{can, t} \Rightarrow d \ln P_{can, t} = d P_{can, t} / P_{can, t}$$

$$d \ln P_{US, t} / d P_{US, t} = 1 / P_{US, t} \Rightarrow d \ln P_{US, t} = d P_{US, t} / P_{US, t}$$

- ◆ Oct. 1991- Jun. 2012 , the data of Canada and the U. S.

Dependent Variable: LNEXCAN

Method: Least Squares

Date: 09/25/17 Time: 01:28

Sample: 1991M10 2012M06

Included observations: 249

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.211777	0.008852	23.92365	0.0000
DP	2.035146	0.234538	8.677266	0.0000
R-squared	0.233621	Mean dependent var	0.241169	
Adjusted R-squared	0.230518	S.D. dependent var	0.147121	
S.E. of regression	0.129054	Akaike info criterion	-1.249168	
Sum squared resid	4.113787	Schwarz criterion	-1.220915	
Log likelihood	157.5214	Hannan-Quinn criter.	-1.237795	
F-statistic	75.29494	Durbin-Watson stat	0.034430	
Prob(F-statistic)	0.000000			

Some inferences (OLS實證結果之分析)

1. $R^2 = 0.2336$

The estimated result of PPP shows that R^2 is 0.2336, and this value is too low, implying the equation of PPP does not cover some important variables to explain the exchange rate changes. (PPP的實證結果顯示判定係數 $R^2 = 0.2336$ ，此數值太低，表示估計式中某些影響匯率變動之重要變數被忽略)

2. The result of coefficient

$$H_0: \beta=0 \quad ; \quad H_1: \beta \neq 0 \quad \Rightarrow \quad t_{\beta} = 8.6772 > t_{0.025} = 1.96$$

$$\text{P-value } (t_{\beta} = 2.0351) = 0.0000 \Rightarrow \text{reject } H_0: \beta=0$$

Inference (推論)- the value of coefficient, $\beta = 2.0351$, is significant at the 5% significant level, which implies that differences in rates of inflation can explain the exchange rate changes, while the sign of β is consistent with the PPP equation. (解釋變數dP之係數值 $\beta = 2.0351$ ，此係數在5%水準下為顯著，顯示「兩國通膨率差距」此變數能解釋匯率之變動，且係數值為正，此符合PPP理論)

3. Examine whether β equals 1 -

$$\ln S_t = \alpha + \beta (\ln P_{Can,t} - \ln P_{US,t}) + \varepsilon_t$$

$$H_0: \beta=1 \quad ; \quad H_1: \beta \neq 1$$

The Wald test' result show that the value of $\chi^2 = 19.4795$ and its p-value is 0.0000 *Reject* $H_0: \beta=1$ that is $\beta \neq 1$

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
t-statistic	4.413558	247	0.0000
F-statistic	19.47950	(1, 247)	0.0000
Chi-square	19.47950	1	0.0000

Null Hypothesis: C(2)=1

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
-1 + C(2)	1.035146	0.234538

Restrictions are linear in coefficients.

- ◆ Although differences in rates of inflation between the two countries can explain the variation in exchange rate changes of CAD/USD, the value of R^2 is too low showing that the equation of PPP does not cover some important variables to explain the exchange rate changes. (美加兩國通膨率的差距(dP)之係數顯著為正，即此變數可以解釋匯率變動，但判定係數 $R^2 = 0.2336$ 太低，顯示PPP此式中未考量某些影響匯率變動之重要變數)
- ◆ Furthermore, the coefficient of differences in rates of inflation does not equal one, $\beta \neq 1$, which imply that CAD exchange rate changes did not move on a one-for-one basis to the inflation differential. (針對兩國通膨率差距(dP)係數等於1($\beta=1$)進行檢定，根據wald檢定結果顯示此係數不等於1，因此，加幣匯率與通膨率差距(dP)兩者間並非1對1的互動關係。)

relative ppp: evidence from long-run data

- ◆ fig4.4 shows that there is a problem of “stationary and nonstationary time series” (時間序列是否為恒定)
- ◆ The tendency for a time series to revert back to its mean is characteristic of a stationary time series.(若時間序列有向平均數修正之特性，則為恒定或定態)
- ◆ A random walk(隨機漫步) is an example of a nonstationary(非定態) times series because it holds no tendency to return to its starting value(or any value).
- ◆ Meese & Rogoff(1988), Mark(1990) have no rejected the possibility that real exchange rates evolve as a random walk without any mean reverting properties.(有些文獻研究結果得到實質匯率為隨機漫步)

➤ Define real exchange rate $Q \equiv \frac{SP_f}{P_d}$.

➤ If PPP is true, Q should be a constant.

(若PPP成立，實質匯率 Q 為一常數，即實質匯率不具有“單根”現象)

=> 可針對實質匯率 Q 之自我迴歸(autoregressive process)來檢驗是否有單根現象

$$Q_t = \delta Q_{t-1} + u_t$$

where $u_t \equiv$ random disturbance
with $E(u_t) = 0$.

$H_0 : \delta = 1 \Rightarrow Q$ 為 random walk(隨機漫步) or unit root(單根)

$H_1 : \delta = 0 \Rightarrow Q$ 為充分向PPP調整 (full adjustment to PPP)

$H_{Alt} : 0 < \delta < 1 \Rightarrow Q$ 為逐漸向PPP調整
(gradual adjustment to PPP).

- When the parameter $\delta = 1$, the real exchange rate evolves as a random walk ($\delta = 1$ ，實質匯率為隨機漫步，也表示實質匯率有單根).
- In the random-walk case, today's exchange rate is the same as yesterday's exchange rate except for the random-error term (匯率為隨機漫步，表示今天的匯率會與昨天匯率相同，除非有意外的隨機干擾)

◆ 何謂單根？

→ $y_t = y_{t-1} + u_t$

→ 隨時漫步(random walk，上式為 u_t 為 i.i.d. 且均數為零)
為單根的特例

- 不論預測多久以後的未來期間或擁有多少的過去的訊息，對於未來的最佳預測為今日的數值
- 隨機漫步的預期值與 t 無關
- 隨機漫步的變異數為時間的線性遞增函數 (nonstationary).

- ◆ Suppose we want to predict the exchange rate one step ahead. This is defined as:

$$Q_{t+1} = \delta Q_t + u_{t+1} \quad (4.4)$$

- ◆ Suppose we want to predict the exchange rate two steps ahead. This is defined as:

$$Q_{t+2} = \delta Q_{t+1} + u_{t+2}$$

$$\Rightarrow Q_{t+2} = \delta(\delta Q_t + u_{t+1}) + u_{t+2}$$

$$\Rightarrow Q_{t+2} = \delta^2 Q_t + \delta u_{t+1} + u_{t+2}$$

.....

- ◆ So in general, the n-step-ahead definition of the exchange rate is:

$$Q_{t+n} = \delta^n Q_t + \delta^{n-1} u_{t+1} + \delta^{n-2} u_{t+2} + \dots + \delta^2 u_t + \delta u_{t+n-1} + u_{t+n}$$

$$E(Q_{t+n}) = \delta^n Q_t$$

$$E(Q_{t+n}) = \delta^n Q_t$$

- $\delta=1$ ，表示實質匯率為隨機漫步，也表示有單根，也表示可以今日之匯率為未來匯率之最佳預測

$$Q_{t+n} = \delta^n Q_t + \delta^{n-1} u_{t+1} + \delta^{n-2} u_{t+2} + \dots + \delta^2 u_t + \delta u_{t+n-1} + u_{t+n}$$

當 $\delta=1$ ，也表示過去的誤差，將會對未來匯率產生永久影響

- 若 $\delta < 1$ ，則 $\delta^n < 1$ ， n 增加 δ^n 會遞減，

=> 未來匯率會向平均值修正，且也意謂著過去的誤差
，隨時間經過對匯率的影響效果↓

- 當 $\delta < 1$ ，匯率為一個向平均數修正的恆定序列

◆ 若匯率為隨機漫步(a random walk)

(1) Any forecast of the future is the same as today's rate,
no matter how far ahead we forecast

(未來匯率預測會等於今天匯率)

(2) The exchange rate has no tendency to return toward its
(prior) mean value. (匯率不會向均數修正)

*The random walk is an example of a nonstationary series.

(隨機漫步為非恒序列的一例)

Testing PPP

$$\underline{Q_t} = \alpha + \beta \underline{Q_{t-1}} + \underline{e_t}$$

If $\beta = 1 \Rightarrow$ unit root (單根)

- 1980年代實證，許多結果顯示無法拒絕 $H_0 : \beta = 1$ ，即匯率有單根現象
- 原因：
 - **Insufficient power** in the tests, due to insufficient data. Since 1990, studies have sought more data.
 - However, likely reflects low **power** (原因為樣本不足，使檢定力不足，因而得到錯誤推論)

<i>Studies with long time series:</i>	Time Period	Estmt. δ	Speed of Adjustment	Half-life (years)
JF (1990)	1869-1987	.84	.16	4
Updated WTP (2007)	1791-2005	.88	.12 (s.e.=.05)	4
Lothian & M. Taylor (1996)	2 centuries			3-5
Alan Taylor (2002) 19 currencies	1870-1996	.79	.21 (s.e.=.01)	3.4 – 4.1
Pappell & Prodan (2005)	1870-1998			1-2

- Use longer time series (拉長樣本期間有制度改變的問題)
But span different exchange rate regimes
- Use panel data (使用追蹤資料)
Apply unit root tests to panels of countries

How do we test for a unit root?

- ◆ Dickey and Fuller (1976, 1979).

$$y_t = \phi y_{t-1} + u_t$$

H_0 : series contains a unit root ($\phi = 1$)

H_1 : series is stationary. ($\phi < 1$)

- ◆ We usually use the regression:

$$\Delta y_t = \psi y_{t-1} + u_t$$

$$\phi - 1 = \psi$$

We test $H_0: \psi = 0$ vs. $H_1: \psi < 0$.

If H_0 is rejected $\Rightarrow y_t$ does not contain a unit root.

The Augmented Dickey Fuller Test (ADF)

- 當被解釋變數 Δy_t 有自我相關，會使殘差產生自我相關問題，此時可增加 Δy_t 的落後期，故稱為擴大型DF檢定(ADF))

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^p \alpha_i \Delta y_{t-i} + u_t$$

$H_0: \psi=0$ vs. $H_1: \psi<0$.

H_0 is rejected $\Rightarrow y_t$ does not contain a unit root.

- 若將非定態變數經差分，可使其達到恆定狀態；若一時間序列經一次差分而獲得穩定時間序列，稱此序列整合階次（integrated of order）為一，若經 d 次差分後才達到穩定情況，則稱此數列整合階次為 d ，記為 $y_t \sim I(d)$ 。通常總體經濟時間數列資料多為 $I(1)$ 序列。

Example: PPP

	Variables	Frequency	Sources
Currency exchange rate	ls=Log (S)	Annual (1979-1990)	Hayashi (2000)
Price index of UK	lukwpi=log (ukwpi)		
Price index of US	luswpi=log (uswpi)		

- ◆ Real exchange rate (將實質匯率取對數 $\log(\frac{SP_f}{P_d})$)

$$e_t = ls_t - luswpi_t + lukwpi_t$$

Non-stationary Time Series

◆ Unit Root Test

→ ADF Test

$$\tau : \Delta Y_t = \beta Y_{t-1} + \sum_{i=1}^p \delta_i \Delta Y_{t-i} + \varepsilon_t \quad \longrightarrow \quad \text{De-data}$$

$$\tau_t : \Delta Y_t = \alpha + \gamma t + \beta Y_{t-1} + \sum_{i=1}^p \delta_i \Delta Y_{t-i} + \varepsilon_t \quad \longrightarrow \quad \text{De-trend}$$

$$\tau_u : \Delta Y_t = \alpha + \beta Y_{t-1} + \sum_{i=1}^p \delta_i \Delta Y_{t-i} + \varepsilon_t \quad \longrightarrow \quad \text{De-mean}$$

→ KPSS

$$Y_t = \eta t + r_t + \varepsilon_t \quad \varepsilon_t \stackrel{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

Non-stationary Time Series

♦ Selection Criteria of the Lag Length

→ Schwartz Bayesian Criterion (SBC)

$$\min SBC = \ln\left(\frac{SSR}{T}\right) + \frac{k \ln T}{T} \rightarrow \text{Small sample}$$

→ Akaike Information Criterion (AIC)

$$\min AIC = T \ln\left(\frac{SSR}{T}\right) + 2k \rightarrow \text{Big sample}$$

k → parameters

T → observations

SSR → sum of squared residuals

➤ 以加幣兌美元的實質匯率(RE)進行單根檢定，若RE無單根存在，表PPP成立；反之，若RE有單根存在，表示PPP不成立。

- ◆ Unit root test of Real exchange rate (CAD/US\$)
- ◆ ADF without trend(level)

Null Hypothesis: RE has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic - based on SIC, maxlag=6)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.504928	0.5297
Test critical values:	1% level	-3.456622	
	5% level	-2.872998	
	10% level	-2.572951	

*MacKinnon (1996) one-sided p-values.

◆ Unit root test of Real exchange rate ADF with trend(level)

Null Hypothesis: RE has a unit root

Exogenous: Constant, Linear Trend

Lag Length: 0 (Automatic - based on SIC, maxlag=6)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.559056	0.2997
Test critical values:	1% level	-3.995492	
	5% level	-3.428049	
	10% level	-3.137397	

*MacKinnon (1996) one-sided p-values.

- ◆ Unit root test of Real exchange rate (CAD/US\$)
- ◆ ADF without trend(difference)

Null Hypothesis: D(RE) has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic - based on SIC, maxlag=6)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-17.12446	0.0000
Test critical values:	1% level	-3.456730	
	5% level	-2.873045	
	10% level	-2.572976	

*Mackinnon (1996) one-sided p-values.

- ◆ Unit root test of Real exchange rate (CAD/US\$)
- ◆ ADF with trend(difference)

Null Hypothesis: D(RE) has a unit root

Exogenous: Constant, Linear Trend

Lag Length: 0 (Automatic - based on SIC, maxlag=6)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-17.33613	0.0000
Test critical values:		
1% level	-3.995645	
5% level	-3.428123	
10% level	-3.137440	

*Mackinnon (1996) one-sided p-values.

- ◆ Unit root test of Real exchange rate (CAD/US\$)
- ◆ PP without trend(level)

Null Hypothesis: RE has a unit root

Exogenous: Constant

Bandwidth: 7 (Newey-West automatic) using Bartlett kernel

	Adj. t-Stat	Prob.*
Phillips-Perron test statistic	-1.526959	0.5184
Test critical values:		
1% level	-3.456622	
5% level	-2.872998	
10% level	-2.572951	

*MacKinnon (1996) one-sided p-values.

- ◆ Unit root test of Real exchange rate (CAD/US\$)
- ◆ PP with trend(level)

Null Hypothesis: RE has a unit root

Exogenous: Constant, Linear Trend

Bandwidth: 6 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-2.546372	0.3056
Test critical values:	1% level	-3.995492	
	5% level	-3.428049	
	10% level	-3.137397	

*MacKinnon (1996) one-sided p-values.

- ◆ Unit root test of Real exchange rate (CAD/US\$)
- ◆ PP with trend(difference)

Null Hypothesis: D(RE) has a unit root

Exogenous: Constant

Bandwidth: 6 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-17.06433	0.0000
Test critical values:	1% level	-3.456730	
	5% level	-2.873045	
	10% level	-2.572976	

*MacKinnon (1996) one-sided p-values.

- ◆ Unit root test of Real exchange rate (CAD/US\$)
- ◆ PP with trend(difference)

Null Hypothesis: D(RE) has a unit root

Exogenous: Constant, Linear Trend

Bandwidth: 6 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-17.28313	0.0000
Test critical values:	1% level	-3.995645	
	5% level	-3.428123	
	10% level	-3.137440	

*MacKinnon (1996) one-sided p-values.

Results of unit-root tests for real exchange rate

	Model	ADF		PP	
		T-Statistic (lag)	P value	T-Statistic (bandwith)	P value
Level	Without trend	-1.5049 (0)	0.5297	-1.5269 (7)	0.5184
	With trend	-2.5590 (0)	0.2997	-2.5463 (6)	0.3056
First Difference	Without trend	-17.1244*** (0)	0.0000	-17.0643*** (6)	0.0000
	With trend	-17.3361*** (0)	0.0000	-17.2831*** (6)	0.0000

Notes: The lag lengths are determined via the SIC and are in parentheses. The bandwiths are for the Newey-West method of the PP tests in parentheses.

- ◆ The results of table show that real exchange rate of CAD/US\$ follows I(1) process, where a unit root is existing in each level of real exchange rate. Hence, the result implies that PPP is not true for the case. (由表中結果顯示加幣兌美元之實質匯率為I(1)過程，即實質匯率水準值有單根，因此PPP不成立。)